# Estimating dynamic diversion ratios in storable good industries 

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June 2023

This paper estimates diversion ratios capturing the influence of dynamic substitution patterns on forward-looking storable goods firms' profits. These dynamic diverted value ratios are key inputs in a new dynamic upwards price pressure test, dGUPPI. This new method obviates the need to estimate consumers' dynamic demand functions and is computable within a policy-making timeframe. To illustrate its practical use to policymakers, it is applied to the UK laundry detergent industry from 2002 to 2012. Estimated bounds on the dynamic diverted value ratios calculate set-valued dGUPPI that overturn standard static empirical policy tools that would incorrectly permit an anticompetitive hypothetical merger.

Keywords: dynamic demand, dynamic pricing, storable goods, price elasticities, diversion ratios, GUPPI

JEL Classification: D12, C54, L11, L40, L67

[^0]
## 1. Introduction

Incorporating dynamics into demand models enables an analyst to accurately measure competitive interactions in storable goods industries. Static demand models do not allow a portion of short-run volume increases from temporary price cuts to draw down on future sales. As a result, own price elasticities are overstated and cross-price elasticities tend to be understated. The result is severely biased value and volume diversion ratios - key inputs into empirical policy analysis.
Consistent estimates of demand responses that capture both contemporaneous and inter-temporal substitution are difficult to obtain in practice. This is because estimation of sufficiently flexible dynamic demand models for storable goods is resource intensive, constrained by industry features, and often have stringent data requirements. As such, they are unlikely to be feasible within a policy-making time horizon. This paper develops a new computationally light method to estimate diverted value ratios that capture the effect that storable good demand dynamics have on firms' pricing incentives. ${ }^{1}$
This new empirical method shares some key characteristics of Hendel and Nevo's (2006) dynamic demand estimation for storable goods. In both cases, a three-step estimation procedure is used to quantify long-run demand responses to price changes. Moreover, in the first two steps, a (biased) static demand model and a price process are estimated. The most pronounced differences arise in the last step of the estimation procedures.
In their final step, Hendel and Nevo (2006) repeatedly solve the consumer's dynamic program while estimating the remaining demand parameters. Then, to simulate each long run price elasticity, a new price process is estimated and the consumer's dynamic program is re-solved. Consequently, estimation of the dynamic demand model and simulation of long-run demand responses can be computationally demanding.
In contrast, the final step of this new approach estimates dynamic diverted value ratios using the pricing equations from a dynamic supply model. In the pricing equations, the effect that consumers' inter-temporal substitution has on firms' pricing decisions are captured by reducedform parameters - labelled 'dynamic correction ratios'.
Empirical analogues of the pricing equations are created by populating them with the observed sequence of market outcomes, the estimated price process, and the output of the estimated static demand model. The only remaining unknown components are the dynamic correction ratios.
Estimates of these reduced-form parameters are obtained by equating the firm's margin estimated directly from its (internal) accounts with its counterpart in the dynamic supply model's empirical pricing equations. Finally, a simple calculation uses estimated dynamic correction ratios to convert short-run diverted value ratios into their dynamic counterparts.

[^1]This new approach has several practical benefits for policy analysts. First, the estimation can be completed within the timeframe of the policy making process. This is because, unlike many existing dynamic demand estimation methods for storable goods, the cost of solving the consumer's dynamic program is avoided in estimation and simulation. It is replaced by a less computationally intensive and easier to implement root-finding step.
Second, the new method places fewer requirements on idiosyncratic features of the storable industry than existing dynamic demand methods (i.e. no restriction on the number of product sizes or overall number of products). As such, analysts can apply this approach to a large number of storable good industries.
Third, the analyst only needs to have access to market data matching the price setting frequency (i.e. weekly) and margins measured over the same period. ${ }^{2}$ This is often available to policy analysts through information supplied by firms as part of the policy process.
Finally, dynamic diverted value ratios, like their static counterparts, are shown to be key inputs into a new generalised upward price pressure index adjusted for demand dynamics (dGUPPI). This new empirical policy tool extends the existing GUPPI test and can be used to evaluate mergers in industries with dynamic demand.
However, the ease of implementation and wider applicability of this new approach come at the cost of a loss of point identification: estimates of dynamic diverted value ratios are setvalued. Consequently, counterfactuals and policy tools that use these as inputs produce a set of outcomes, rather than a single value. However, as illustrated in this paper's application to the UK laundry detergent industry, set-valued results of empirical policy analysis need not reduce their efficacy in a policy setting.
This new approach uses a dynamic model of demand and supply for storable good industries. ${ }^{3}$ In the supply model, setting optimal prices requires that firms solve an infinite horizon high-dimensional dynamic programming problem with continuous controls and states. However, as highlighted by Rust (2019), limited by information processing capabilities and cognitive constraints, firms cannot precisely solve such problems. Some form of approximation is necessary.
Noting that inter-temporal substitution predominantly draws forward purchases from the near future, in this paper firms are assumed to use a finite multi-period lookahead rolling horizon procedure when setting prices. This approach is aligned with the existing literature in approximate dynamic programming (Powell (2011); Bertsekas (2011)), reinforcement learning

[^2](Bertsekas (2021)), and is in line with empirical evidence on the behaviour of forward-looking firms (Che et al. (2007); Kunz et al. (2023)). Since neither consumers nor firms have the resources to meet the unbounded rationality requirements of the default equilibrium concept Markov Perfect Equilibrium - Experience Based Equilibrium (EBE) (Fershtman and Pakes (2012)) is used.

To illustrate how this method can be employed in practice it is applied to the UK laundry detergent industry. The application uses Kantar Worldpanel purchase diary data collected between 2002 and 2012 and published accounts to estimate firm margins for each calendar year. ${ }^{4}$ The three-step procedure briefly outlined above is used to estimate bias-scaled dynamic correction ratios in each year for the two firms, A and B, that dominate the UK laundry detergent industry.
In the first step, the analyst specifies how price setting decisions are fed back into the internal price forecasting processes. If the analyst has evidence that the firm treats price forecasts as fixed when setting prices then the expected future price derivatives are zero. With this open loop forecast assumption, no further action is required in step one because expected changes in the evolution of future prices do not directly affect price setting. However, if the analyst believes that there is a feedback loop between price setting and forecasting, then closed-loop model of price setting and forecasting is needed.
In a closed-loop scenario a statistical model of the price process is estimated and used to construct expected price derivatives. For the UK laundry detergent industry, a dynamic factor model is estimated from observed prices for this purpose. The reasons are three-fold. First, it is compatible with high-dimensional forecasting. Second, it can flexibly capture observed productspecific price dynamics. Finally, estimation of high-dimensional expected price derivatives requires only basic linear algebra operations. ${ }^{5}$
In the second step of the estimation procedure, the analyst estimates a standard static demand model (i.e. logit, nested logit, mixed logit) using high-frequency market data matching the price setting frequency (i.e. weekly). The demand model can be estimated using aggregate market data or consumer level micro-data.
The reduced form parameters that capture the effect of demand dynamics on firm's pricing incentives are computed in the final step. In the empirical model, these reduced form parameters are bias-scaled dynamic correction ratios. ${ }^{6}$ As noted above, they are computed by setting the expression for the margin from the dynamic supply model equal to its empirical counterpart

[^3]derived from firm's accounts. Because the margin estimated using firm accounts is likely to span multiple time periods and products, the price equations in the model are summed over same products' and time periods. Once estimated, own and cross-price bias-scaled dynamic correction ratios for any two products can be used to translate the corresponding (downward biased) short-run diverted value ratio into a dynamic diverted value ratio. ${ }^{7}$
However, with more than a handful of products, the values of the bias-scaled dynamic correction ratios that equate the empirical and model margin belong to sets whose bounds are likely to be too wide to be useful in a policy setting. To remedy this, further identification restrictions are imposed.
First, attention is restricted to the average bias-scaled dynamic correction ratio measured over the accounting period covered by the margin. Second, the overstatement of short-run own-price demand responses are constrained to be the same for all products whose sales contribute to the margin. Similarly the understatement of short-run cross-price demand responses are also assumed to the same across the products included in the margin. Finally, a data-driven choice of the upper bound on the cross-price dynamic correction ratio is imposed.
These additional constraints reduce the number of parameters to be estimated to two. With a single equation and two unknown parameters, the final step of the estimation procedure searches for all values of the pair of bias-scaled dynamic correction ratios that equate the model and accounting estimate of the margin.
Using the resulting bounds on dynamic diverted value ratios for the two dominant firms in the UK laundry detergent industry, I explore the effect that innovation resulting in product compaction - shrinking dosage per wash - has on bias when using mis-specified static demand models. ${ }^{8}$ As more detergents are compacted, physical pack sizes shrink without reducing the number of washes. In turn, reducing inventory holding costs. With convex inventory costs, product compaction leads to a relative increase in the demand for large pack-sizes. The larger amount of future consumption purchased in one store visit, the larger the correlation between inventories and prices. As such, the bias due to the omission of demand dynamics increases with the product innovation. In line with this, I find that diversion ratio biases in the UK laundry detergent industry increase during the phase of intensive product compaction.
To show how the dynamic diverted value ratios can be used in a policy setting, I use a hypothetical brand acquisition to show how a set-valued dGUPPI can inform the likelihood that harmful unilateral effects arise from changes in industry structure. I find that conducting the policy analysis without accounting for market dynamics can lead to policy errors. In the experiment without accounting for demand dynamics, the brand merger would be incorrectly permitted.

[^4]Related literature This paper contributes to the nascent literature examining how firms facing solve complex high-dimensional dynamic decision problems in practice (Rust (2019); Iskhakov et al. (2020); Hortaçsu et al. (2023); Che et al. (2007); Kunz et al. (2023)). Specifically, this paper draws on techniques and insights from approximate dynamic programming (Powell (2011); Bertsekas (2011)), reinforcement learning (Bertsekas (2021)), high-dimensional forecasting (Bai and Ng (2002); Stock and Watson (2002)), and workhorse IO static demand system estimation (Berry (1994); Berry et al. (1995); Compiani (2022)) to exploit information on market power imparted using promotional pricing strategies over a prolonged period to calculate bounds on dynamic diverted value ratios in the UK laundry detergent industry.
It also contributes to the literature on dynamic demand estimation for storable goods. Following seminal papers by Erdem et al. (2003) and Hendel and Nevo (2006), there have been several papers that have sought to apply and extend the frameworks they develop. For example, Pires (2016); Wang (2015); Osborne (2018); Crawford (2018) estimate a variety of extensions of these dynamic demand models using panel micro-data). It is also related to Hendel and Nevo (2013) and Perrone (2017). In contrast to other papers in this field, they develop models that can quantify the effect of omitting consumer dynamics on estimates of long-run price elasticities without incurring the substantial computational burden of solving for the consumer's value function.
Like Perrone (2017) the approach developed in this paper is straightforward to implement and is flexible in terms of consumer heterogeneity and price expectations. It also builds on Perrone (2017) by allowing for product differentiation and quantity discounts within a storable good industry.
Hendel and Nevo (2013) develop a simple dynamic demand model for storable goods. Like the framework developed in this paper, their model can be estimated using market level data. However, it requires more restrictive assumptions on consumer storage technology, taste heterogeneity and price expectations to do so. Moreover, and importantly from a policy perspective, it is challenging to scale up the model Hendel and Nevo (2013) propose beyond a handful of products. In contrast, the output of the model in this paper naturally scale to high-dimensional choice sets that are often observed in storable good industries.
The cost of the flexibility of the approach in this paper - the loss of point identification - is not shared by other approaches. However, this need not reduce the efficacy of policy analysis based on its outputs. This is demonstrated using a new dynamic version of the price pressure test, dGUPPI, developed in this paper. This new price pressure test extends the set of tools available to antitrust practitioners assessing mergers exhibiting price dynamics.

Finally, it also contributes to a literature analysing dynamic demand and supply models for durable goods - of which storable goods are a special case. The most closely related are Chen et al. (2008). They use data from a simulated dynamic demand and supply model of the used car market to explore the biases in price elasticities and corresponding market power estimates
when a misspecified static demand model is used instead of a dynamic demand model. ${ }^{9}$

Outline The remainder of the paper is structured as follows. Section 2 provides an overview of the UK laundry detergent industry. Section 3 describes a dynamic demand and supply side model of the industry. Using the price setting equations from the model, section 4 describes the three-step empirical procedure and additional identification assumptions that produce bounds on dynamic correction ratios. Section 5 contains the estimation results. Section 6 introduces dynamic GUPPI and shows how dynamic diverted value ratios are used to compute it. Its use in a policy setting is illustrated using a hypothetical merger. Section 7 concludes.

## 2. UK Laundry detergent

This paper focuses on estimating dynamic diverted ratios that capture competitive intensity for the UK laundry detergent industry - the storable good studied in the US by Hendel and Nevo (2006).

Data The analysis of the UK laundry detergent industry is based on individual household purchase data from 1st January 2002 until 31st October 2012. Households that take part in the survey scan the barcode of the items they purchase. Using the scanned barcode, the survey records the price and number of packs bought together with the characteristics of the product purchased. In addition, the purchase date and store in which the product was bought is also recorded. The purchase data is supplemented by annually updated household demographics.
Laundry detergent - like many other storable goods - is a mature industry. The majority of sales occur in a few large, national supermarket/specialist retail chains. Prices charged by UK supermarkets are the same for all of their stores if their footprint is greater than 280 sqft. ${ }^{10}$ To avoid complexities related to store choice, the empirical analysis is focuses on purchases from a leading UK supermarket chain.

Industry Overview The manufacture of UK laundry detergent is (approximately) a duopoly: two firms account for around 75 to 85 percent of households' annual purchases in each year between 2002 and 2012. Hereafter, they are referred to as firm A and firm B. Firm A's annual share of sales is between 44 and 54 percent and firm B is between 29 and 36 percent. Outside of these two major producers of branded products, the retailer's private label products commands the largest share - although its share has declined from 25 percent in 2002 to 12 percent in 2012. A fringe of small niche brands account for the remainder of products sold.

[^5]Laundry detergent is sold in a diverse array of brands. In addition to retailer's private label products and the fringe of 'other' brands, the two largest firms sell the six major brands between them. These six brands are labelled alphabetically as brand A through to brand F and are differentiated by their perceived quality. ${ }^{11}$ Using the average purchased price per wash as an indicator for consumer perception of quality, brand $D$ is classified as a premium brand (20p per wash). Brands A, C and E are mid-range brands (17-18p per wash) and brands C and F are 'standard' brands whose price is similar to the supermarket's private label detergents (13-14p per wash).
Laundry detergent brands are available in different formats: powder, tablets, liquids, liquid capsules, super concentrated liquid, and gel. They have different dosage metrics: liquid and gel are measured in milliliters per wash, whereas powder and tablets are measured in grams per wash.
Each laundry detergent is defined by its format, its brand and the chemical properties of the enzymes it contains (i.e. non-bio/bio, stain removal properties, scent etc). In this paper, given similar pricing patterns, a detergent is defined by its brand and format. A product (or pack), in addition to the detergent it contains, is defined by the number of washes in contains (i.e. pack size).
Table 1 provides summary statistics of the 668 laundry detergent products sold over the sample period together with consumer characteristics (including household size and a proxy for household income - i.e. average weekly grocery spend).

Table 1: Summary of Kantar Worldpanel data

|  | Mean | Median | Std dev | Min | Max |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Purchase Characteristics |  |  |  |  |  |
| Price in (£) | 3.58 | 3.20 | 1.81 | 0.13 | 24.52 |
| Price per wash (£) | 0.17 | 0.16 | 0.06 | 0.01 | 0.77 |
| Purchased Washes | 23 | 20 | 12 | 3 | 100 |
| Dosage in Grams | 87.7 | 80 | 16.1 | 18.3 | 139.1 |
| Dosage in Millilitres | 52.6 | 45 | 20.8 | 12 | 125 |
| Number of Packs | 1.07 | 1 | 0.26 | 1 | 2 |
| Household Characteristics |  |  |  |  |  |
| Number of Equivalent Adults | 2.28 | 2.20 | 0.79 | 1.00 | 8.10 |
| Av. weekly grocery spend $(£)$ | 67.05 | 63.78 | 27.71 | 3.70 | 423.45 |

Source: Kantar Worldpanel

Product Compaction Following a series of industry-wide initiatives from 2002 to 2005 that sought to reduce the environmental impact of detergent production, products were 'compacted'

[^6]Table 2: Inter-purchase duration in weeks

|  | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Median | 3 | 4 | 4 | 4 | 4 | 4 | 4 | 5 | 5 | 5 | 5 |
| Mean | 5 | 6 | 7 | 7 | 7 | 7 | 8 | 9 | 9 | 10 | 10 |

Source: Kantar Worldpanel. Note: To compute inter-purchase duration, consumer purchases across all retailers are included.
and dosages reduced over time. As a result, the physical storage space occupied by the material needed per wash also decreased.
The compaction process accelerated after the new formats - super concentrated liquid and gel - were brought to market in 2007 and 2008, respectively. In response, firms gradually produce larger pack sizes over time. ${ }^{12}$ The average number of washes per purchased pack have increased by around 50 percent from 17 washes in 2002 to 26 washes in $2012 .{ }^{13}$
With larger products and reduced dosage per wash, detergent compaction facilitates longer periods of consumption from inventory. Table 2 presents the median and mean inter-purchase durations over the sample period. The median time between purchases grows from three weeks in 2002, to four weeks from 2003 to 2008, and five weeks from 2009. This increase in interpurchase duration over time is consistent with the ability of consumers to take advantage of product innovation reducing storage cost per wash and buy larger products.
During this period, inter-temporal substitution is likely to become a larger component of overall substitution. All else equal, the bias from omitting demand dynamics might be expected to increase. In addition to estimating dynamic correction ratios and dynamic diverted value ratios, this paper explores the effect that product compaction has on the bias arising when dynamics are omitted from demand estimation.

## 3. Price setting with demand dynamics

This section presents a model of the UK laundry detergent industry in which both consumers and firms are forward-looking. When setting current prices, firms take demand and supply dynamics into account. In each period firms set price to maximise their perception of the present value of expected profit flows.

[^7]When there are no inter-temporal links in demand or costs, the firm's optimisation problem is separable and is solved independently in each time period. However, laundry detergent demand is inherently dynamic. For example, if the firm discounts its product(s) as part of a temporary promotion, consumers may accelerate purchases and stock the product at home for future consumption. When demand is dynamic, firms consider the effect that current prices have on demand today and in the future. The effect of a price change today on current and expected future profits are affected by static and inter-temporal substitution.
When setting prices, firms form beliefs about both consumer and firm dynamics. Therefore, firms must retain and process a lot of information to compute consumer demand and assess the profitability of a very large number of pricing strategies. The size of the task of determining optimal pricing strategies is exacerbated by the large number of products sold to consumers in storable good industries (i.e. often the choice set contains $50-150$ products).
Faced with these large computational and cognitive challenges, firms are only able to solve an approximate version of their dynamic pricing problem. In line with this approach, the equilibrium concept used in the resulting high-dimensional dynamic pricing game is Experience Based Equilibrium (Fershtman and Pakes (2012)). This allows agents to have imperfect information about other industry participants and relaxes stringent rationality requirements. Agents' play need only maximise payoffs given the information available to them and be consistent with outcomes in states they have previously experienced.

This section is structured as follows. First, consumer demand and firms' empirical aggregate dynamic demand function is described. Second, the dynamic supply model is presented and recast in terms of reduced form parameters - dynamic correction ratios - formed by dividing inter-temporal diverted value ratios by static diverted value ratios. They capture omitted demand dynamics and can be used to construct the present value of contemporaneous and inter-temporal diverted sales. Next, the concept of a 'dynamic diverted value ratio' and their relationship to the dynamic correction ratios is introduced. Finally, the equilibrium concept consistent with the firm's approximations is discussed.

Hereafter, bold symbols indicate vectors and matrices.

### 3.1. Dynamic demand

In each period, consumer's purchasing laundry detergent face a high-dimensional discrete-continuous dynamic optimisation problem. First, they choose whether or not to purchase a pack of laundry detergent - defined as a combination of brand, format, and number of washes it contains. When choosing whether or not to purchase laundry detergent, consumers choose one product from a choice set containing $J$ differentiated products in each week, $t .{ }^{14}$

[^8]At the start of each time period, consumer $i$ privately observes their existing inventories $\boldsymbol{I}_{i t}=$ $\left[I_{i 1 t} \ldots, I_{i J t}\right]^{\top}$ where $I_{i k t}$ is the number of washes of detergent $k$ in stock at the beginning of week $t$. They also observe $J$-vectors of current prices, $\boldsymbol{p}_{t}$, purchase demand shocks, $\boldsymbol{\xi}_{t}$, seasonal demand shifters, $\boldsymbol{y}_{t}$, and any additional information used to forecast future prices, $\boldsymbol{f}_{t}$. Using this information, they assess the present value of their expected utility for each product in their choice set. The utility maximising laundry detergent product is purchased if its utility exceeds that of making no purchase.
Second, the contents of the purchased detergent are stored in inventory and is available for use for consumption in week $t .{ }^{15}$ Unused detergent is carried over in inventory (at cost), ready for consumption in the next period. '
The probability that consumer $i$ purchases detergent $j$ in week $t$ is a function of the consumer's existing inventories, characteristics, $\boldsymbol{\eta}_{i}$, current prices and the vector of non-price state variables, $\boldsymbol{x}_{t}:=\left[\boldsymbol{\xi}_{t}, \boldsymbol{y}_{t}, \boldsymbol{f}_{t}\right]$,

$$
\begin{equation*}
\operatorname{Pr}\left(d_{i t}=j\right)=s_{j}\left(\boldsymbol{I}_{i t}, \boldsymbol{p}_{t}, \boldsymbol{x}_{t}, \boldsymbol{\eta}_{i}\right) \tag{1}
\end{equation*}
$$

where $d_{i t} \in\{0, \mathcal{J}\}$ indicates their chosen option. Aggregate dynamic demand for detergent $j$ in week $t$ is formed by integrating over individual demand functions over the conditional distribution of household inventories and characteristics, $F\left(I_{i t}, \eta_{i} \mid p_{t}^{\mathcal{H}}, x_{t}^{\mathcal{H}}\right)$,

$$
\begin{equation*}
q_{j}\left(\boldsymbol{p}_{t}, \boldsymbol{\omega}_{t}\right)=M_{t} \int s_{j}\left(I_{i t}, p_{t}, x_{t}, \eta_{i}\right) d F\left(I_{i t}, \eta_{i} \mid p_{t}^{\mathcal{H}}, x_{t}^{\mathcal{H}}\right) \tag{2}
\end{equation*}
$$

where $\boldsymbol{\omega}_{t}:=\left[\boldsymbol{x}_{t}, \boldsymbol{p}_{t}^{\mathcal{H}}, \boldsymbol{x}_{t}^{\mathcal{H}}, M_{t}\right]$ is the state-vector excluding current prices and $M_{t}$ is the market size in week $t$.
Equation (2) expresses aggregate dynamic demand implicitly as a function of current prices, past prices, non-price state variables and their histories. As such, a firm - even though they do not observe inventories or consumption - can use their knowledge of purchase and consumption functions together with data on historical market outcomes to evaluate an empirical version of this aggregate dynamic demand function for each detergent $j$ in week $t .^{16}$
In the subsequent supply model, as is common in the empirical (IO and marketing) literature (Nair (2019)), the empirical aggregate dynamic demand functions are assumed to be known to firms when setting prices. ${ }^{17}$

[^9]
### 3.2. Firms

This section describes the dynamic pricing problem facing manufacturers of UK laundry detergents. As noted in section 2, the manufacture of laundry detergent sold in the UK is a duopolistic, mature industry. The majority of sales occurring in a few large, national supermarket/specialist retail chains. Manufacturers offer a large number of products. The set of products tends to evolve slowly over time. As such, the model abstracts away from entry and exit of firms and products.
Without direct information on vertical arrangements and give the upstream duopoly, I assume manufacturers set retail prices. As a result, the retailer's mark-up is implicitly included in a composite marginal cost for a price-setting manufacturer. ${ }^{18}$ This represents a simplification of the complex negotiations between manufacturer and retailer that occur in practice. Extensions considering richer vertical contracting arrangements is left for future study with richer data sources.

Setup Let $n=1, \ldots, N$ index firms in the industry sharing a common discount factor, $\delta \in$ $(0,1)$. The expected discounted sum of per-period profits over an infinite horizon,

$$
\begin{equation*}
\pi_{n t}^{N P V}=\mathbb{E} \sum_{h=t}^{\infty} \sum_{j \in \mathcal{J}_{n}}\left(p_{j h}-m c_{j h}\right) q_{j}\left(\boldsymbol{p}_{h}, \boldsymbol{\omega}_{h}\right) \tag{3}
\end{equation*}
$$

where set of $J_{n}$ products manufactured by firm $n$ is denoted $\mathcal{J}_{n}$ and $m c_{j t}$ is the marginal cost of production for product $j$ in period $t$. Hereafter, let $\boldsymbol{m}_{t}:=\boldsymbol{p}_{t}-\boldsymbol{m} \boldsymbol{c}_{t}$ be the $J$-vector of mark-ups.
Firm $n$ maximises $\pi_{n t}^{N P V}$ by setting a sequence of a prices, $\left\{\boldsymbol{p}_{n h}\right\}_{h=t}^{\infty}$. To set prices firm $n$ solves a high-dimensional dynamic programming problem (DP) for each of the products it sells. In each case it chooses a vector of continuous controls as a function of a high-dimensional and continuous industry state vector. This is a cognitively challenging, resource intensive task for firms - especially given the high frequency of price setting and limited resources they have at their disposal.
Rust (2019) discusses the existing results from the computational science literature linking the problem solving resources of agents and their ability to find optimal solutions to dynamic

[^10]programs as a function of their complexity, size, and nature. He highlights Chow and Tsitsiklis' (1989) result that agents with finite cognitive and computational ability cannot compute the solution to large, complex realistic dynamic problems - like the firm's dynamic pricing problem. Rust (2019) also reviews the nascent literature of case studies investigating the optimality of choices made by firms facing dynamic profit optimisation problems and finds that this appears to be the case across many industries. Among others, these include airlines (Williams (2022)), hotels (Cho et al. (2018)), and car rental companies (Cho and Rust (2008)).
In these case studies firms' strategic choices (i.e. prices, asset replacements) are compared to those that solve the model of the firms' dynamic profit optimisation problems. Against the benchmark optimal solutions, firms' strategic choices are 'nearly optimal', but may leave room for improvement. Hortaçsu et al. (2023) also document how organisational and pricing inefficiencies in a large US airline can lead to internal procedures result that may result in "second-best" pricing policies.
The consistent message from these case studies is that firms facing a large, complex dynamic profit optimisation problems are boundedly rational. That is, they 'satisfice' and use approximations to optimal solutions in practice.

ADP Solutions to complex dynamic programs can be approximated in many ways. Approximate dynamic programming (ADP) encompasses a wide variety of techniques from a collection of disparate fields that have developed specific approaches to approximate solutions to complex dynamic programs they encounter. ${ }^{19}$ Powell (2011) highlights that they have been successfully employed in many industries including transportation, energy, health and finance. Instead of attempting to solve intractable, high-dimensional infinite horizon problems Powell (2011) notes that "a natural approximation is to try to solve the problem over a shorter horizon". Under this ADP approach, boundedly rational firms lookahead a short number of periods ahead, $H$, then solve for optimal policies given this optimisation horizon. They then implement the optimal policy for period $t$ and move forward to period $t+1$. In the next period, they repeat this rolling horizon procedure looking ahead $H$ periods into the future. ${ }^{20}$
Laundry detergent firms have a natural horizon linked to consumer purchase cycles that drives demand dynamics (i.e. purchase, store, consume, purchase). Because consumers respond to promotions by accelerating purchases from the near future, inter-temporal substitution takes place over a short time window. Recognising this, firms may choose prices by solving a rolling horizon approximation to their original DP where the $H$-period lookahead window is linked to consumers' inter-purchase durations. Provided that the majority of inter-temporal substitution has taken place once $H$-periods have passed, the loss of accuracy in the resulting dynamic pricing functions is likely to be limited.

[^11]The presence and length of a fixed-window rolling horizon procedure is tested by Che et al. (2007). They find that firms in the US Cereal industry use a short, fixed horizon when setting prices (i.e. not more than two periods ahead). They repeat the analysis for ketchup - a storable good. Once again they find evidence that firms in the US ketchup industry adopt boundedly rational short fixed-window rolling horizon procedure when setting prices. Kunz et al. (2023) also document how in practice, Zalando, optimise prices over a truncated near-term horizon.
In line with discussion above, hereafter firm $n$ is assumed to be boundedly rational and approximates their computationally infeasible DP using a $H$-period lookahead rolling horizon procedure.

Price setting To succinctly describe the set of first order conditions firm $n$ satisfies when solving the approximate DP I introduce some additional notation. First, let $\boldsymbol{\Delta}_{h}^{q}$ denote the matrices of the current and inter-temporal total derivatives of (expected) demand with respect to changes in current prices. Further, let $\Delta_{h}^{p}$ be the corresponding current and inter-temporal total derivatives of price forecasts. The $(j, k)$-th elements of these matrices are given by equations (4) and (5),

$$
\begin{align*}
\Delta_{j k h}^{q} & :=\frac{d q_{k h}}{d p_{j t}} \forall t \leq h \leq H  \tag{4}\\
\Delta_{j k h}^{p} & :=\frac{d p_{k h}}{d p_{j t}} \forall t<h \leq H \tag{5}
\end{align*}
$$

Further, let $\boldsymbol{\Omega}_{n}$ the ownership matrix for firm $n$ whose $(j, k)$-th entry is

$$
\Omega_{j k n}=\left\{\begin{array}{cc}
1 & \text { if } j, k \in \mathcal{J}_{n}  \tag{6}\\
0 & \text { otherwise }
\end{array}\right.
$$

and its diagonal, $\overrightarrow{\boldsymbol{\Omega}}_{n}$, is a $J$-vector encoding the set of products sold by firm $n$.
Using this notation, the system of first order conditions for firm n's approximation to its DP in matrix form is

$$
\begin{equation*}
\boldsymbol{q}_{n t}+\boldsymbol{\Delta}_{n t}^{q} \boldsymbol{m}_{t}+\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \boldsymbol{\Delta}_{n h}^{q} \boldsymbol{m}_{h}+\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \boldsymbol{\Delta}_{n h}^{p} \boldsymbol{q}_{h}=\mathbf{0} \tag{7}
\end{equation*}
$$

where $\boldsymbol{\Delta}_{n h}^{i}:=\boldsymbol{\Omega}_{n} \odot \boldsymbol{\Delta}_{h}^{i}$ for $i \in\{p, q\}, \boldsymbol{q}_{n t}:=\overrightarrow{\boldsymbol{\Omega}}_{n} \odot \boldsymbol{q}_{t}$, and ' $\odot$ ' denotes the Hadamard product. The first and the second terms in equation (7) correspond to the terms that enter the first order conditions when demand is static and firms are myopic. The additional terms contain total derivatives capturing the effect that price changes today have on future quantities and prices, respectively. ${ }^{21}$

[^12]Demand dynamics The second and third terms in equation (7) capture the effect of contemporaneous and inter-temporal substitution on firm $n$ 's pricing incentives, respectively. The effect of contemporaneous substitution on pricing incentives can be measured by the "value of diverted sales" from rival product $k$ to promoted product $j, \Delta_{j k t}^{q} m_{k t}$, or a short-run diverted value ratio,

$$
\begin{equation*}
D R_{j k t}^{\pi}=-\frac{\Delta_{j k t}^{q} m_{k t}}{\Delta_{j j t}^{q} m_{j t}} \tag{8}
\end{equation*}
$$

Short-run diverted value ratios measure the percentage of the contemporaneous profit uplift for the promoted good that are diverted away from rival product $k$ 's current profits. The higher the diverted value ratio, the more intense the contemporaneous competitive interaction is between the two products.
However, for laundry detergent (and other storable goods) inter-temporal substitution also needs to be accounted for to capture the overall competitive intensity. This is because future sales of both the promoted and rival products' are drawn from future demand as a result of the price cut.
Future sales of differentiated, rival products are more distant substitutes of the promoted product than its own than future sales. The cannibalisation of future profits of rival product are, therefore, expected to be less pronounced than future profits of the promoted product. The closeness of dynamic substitution can be quantified by inter-temporal diverted value ratios. They measure the fraction of promoted product $j$ 's immediate profit uplift pulled forward from the present value of future profits diverted away from product $k$.

$$
\begin{equation*}
I T D R_{j k t}^{\pi}=-\frac{\sum_{h=t+1}^{H} \delta^{h-t} \Delta_{j k h}^{q} m_{k h}}{\Delta_{j j t}^{q} m_{j t}} \tag{9}
\end{equation*}
$$

As in the static case, higher $I T D R_{j k}^{\pi}$ indicates closer competitive interactions between current sales of product $j$ and future sales of all substitute products.
When setting prices, firms take into account the fraction of current sales that are cannibalising profits earned on future sales of their products. The effect of demand dynamics on pricing incentives are captured by 'dynamic correction ratios'. The dynamic correction ratio for a pair of products $j$ and $k$ is calculated by dividing the inter-temporal diverted value ratio by the corresponding short-run diverted value ratio.

$$
\begin{equation*}
\Psi_{j k t}:=\mathbb{E}\left[\frac{I T D R_{j k t}^{\pi}}{D R_{j k t}^{\pi}}\right] \tag{10}
\end{equation*}
$$

It measures the expected change of the present value of future profits earned on sales of product $k$ in response to a temporary price change for product $j$ as a fraction of the change in profits earned on current sales of product $k$.

The element-wise product of the $J \times J$ matrix of dynamic correction ratios, $\boldsymbol{\Psi}_{t}$, and vector of the "value of the diverted sales" for firm $n$ 's products is equal to the dynamic demand term in the first order conditions (eq (7)).

$$
\begin{equation*}
\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \boldsymbol{\Delta}_{n h}^{q} \boldsymbol{m}_{h}=\boldsymbol{\Psi}_{t} \odot \boldsymbol{\Delta}_{n t}^{q} \boldsymbol{m}_{t} \tag{11}
\end{equation*}
$$

Restated in this way, the elements in $\Psi_{t}$ are 'bias correction' terms that scale the change in short-run profit term from the static first order conditions to capture the effect that omitted demand dynamics on pricing incentives.
$\boldsymbol{D} \boldsymbol{D} \boldsymbol{R}_{j k}^{\boldsymbol{\pi}}$ Elements of the dynamic correction ratio matrix, can be used to convert static diverted value ratios into their dynamic counterparts. ${ }^{22}$ The dynamic diverted value ratio is shown in equation (12).

$$
\begin{equation*}
D D R_{j k}^{\pi}:=-\frac{\mathbb{E} \sum_{h=0}^{H} \delta^{h} \Delta_{j k h}^{q} m_{k h}}{\mathbb{E} \sum_{h=0}^{H} \delta^{h} \Delta_{j j h}^{q} m_{j h}}=\frac{1+\Psi_{j k}}{1+\Psi_{j j}} D R_{j k}^{\pi} \tag{12}
\end{equation*}
$$

It measures the reduction in the present value of expected profits earned on current and future sales of product $k$ as a fraction of the present value of expected profits earned on current and future sales of product $j$ in response to a temporary cut in the price of $j$.

Price dynamics The fourth term in the system of first order conditions measures the expected change in the present value of revenues due to anticipated adjustments to future prices in response to changes to current prices. Hereafter, for notational convenience let

$$
\begin{equation*}
\mathbf{\Upsilon}_{n t}:=\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \boldsymbol{\Delta}_{n h}^{p} \boldsymbol{q}_{h} \tag{13}
\end{equation*}
$$

be shorthand for the term measuring the effect that firm $n$ 's beliefs over dynamic supply side responses have on its pricing incentives. Firms' beliefs are formed from their repeated experiences of selling products in the industry. They capture product specific promotional price dynamics in each retail chain and are consistent with the observed price data.
However, tracking and forecasting a high-dimensional choice set of products' prices with retail chain specific promotional dynamics is a cognitively challenging and resource intensive task for

[^13]firms. ${ }^{23}$ Even large, well resourced firms face additional practical considerations that makes it prohibitively costly to integrate complex forecasting and dynamic pricing systems. ${ }^{24}$ In light of these challenges, boundedly rational firms may elect to adopt tractable forecasting models and limit feedback between price forecasts and pricing decisions.
Firms facing considerable organisational frictions might adopt an open-loop forecasting model. In an open-loop model, firms takes forecasted price paths as fixed inputs into the process used to set price. The price setters do not know the underlying forecast models, nor are they given updated prices forecasts until after prices in the current period are chosen. As a result, from the price setter's perspective, future prices do not respond to current prices. Therefore, for firm $n$ in each period $t, \boldsymbol{\Delta}_{n t}^{p}=\mathbf{0}$.
Alternatively, more sophisticated firms might employ a closed-loop price forecasting model. In a closed-loop model price setting is assumed to be fully integrated. Proposed changes in prices are fed back into the price forecasting model and the time profile of inter-temporal price derivatives updated.
In practice, the extent to which organisational structure or other frictions affect the feedback loop between prices and forecasts varies across firms. The baseline supply model in the UK laundry detergent assumes firms use a closed-loop forecast model. The open-loop model is retained for sensitivity analysis.

FOC The system of first order conditions in equation (7) can be expressed in terms of the matrix dynamic correction ratios

$$
\begin{equation*}
\boldsymbol{q}_{n t}+\boldsymbol{\Delta}_{n t}^{q} \odot\left(1+\boldsymbol{\Psi}_{t}\right) \boldsymbol{m}_{t}+\boldsymbol{\Upsilon}_{n t}=\mathbf{0} \tag{14}
\end{equation*}
$$

The dynamic correction ratios convert the myopic value of diverted sales into the present value of the time-profile of diverted sales for a forward-looking firm. However, if there are no demand dynamics, then $\boldsymbol{I T} \boldsymbol{D} \boldsymbol{R}_{t}^{\pi}=\mathbf{0}$ and therefore $\boldsymbol{\Psi}_{t}=\mathbf{0}$. Similarly, if firms are myopic, or use an open-loop forecast model, then $\boldsymbol{\Delta}_{n h}^{p}=\mathbf{0}$ for all $h>t$. Without demand and price dynamics, equation (14) collapses to the first order condition of the workhorse static differentiated Bertrand-Nash model of price competition. Equation (14) serves as the basis for the empirical work recovering dynamic versions of diverted value ratios for storable goods.

### 3.3. Equilibrium

The equilibrium concept for the dynamic pricing game is Experience Based Equilibrium (EBE) - a modification of Markov Perfect Equilibrium (MPE) developed by Fershtman and Pakes

[^14](2012). ${ }^{25}$ A EBE retains a Markovian structure of an MPE, but relaxes the restriction that agents are unboundedly rational. ${ }^{26}$
An EBE replaces the MPE rationality requirements with two weaker conditions. First, given the information set used by agents to make decisions, an EBE requires that agents choose actions that maximise their perceived payoffs. The second condition requires that the perceptions held by agents are consistent with play they have observed in the past when the information set recurs.

Both EBE conditions are met by the decisions taken by consumers and firms in the dynamic demand and price models, respectively. In accordance with the first condition, firms and consumers choose actions that maximise their perceived payoffs calculated using a restricted subset of the full information available to them (i.e. limited information of each product's manufacturer). The second condition is also met; firms' aggregate dynamic demand model, their beliefs rivals' price strategies, and consumers' beliefs over future price are all consistent with past market outcomes.
In the context of the model above, one benefit of using EBE as an equilibrium concept is that it relaxes the implausible MPE requirement that consumer and firm beliefs necessarily coincide. Instead, it only requires that these beliefs are consistent with observed play at recurring states of the game.

## 4. Estimation and Identification

This section describes how the first order conditions from the model (eq. (14)) are combined with data on market outcomes and margins to identify and estimate bounds on dynamic diverted value ratios.
The remainder of this section describes a computationally light three-step estimation procedure. In the first step, an approximation to the dynamic price process is estimated and used to construct price forecasts. The second step uses the data on market outcomes to estimate a static demand model. It provides estimates of quantity demanded in each period and the corresponding matrix of short-run demand derivatives. In the final step, bounds on the elements of the 'reduced-form' matrix of parameters capturing demand dynamics are estimated ready for use in empirical policy analysis.
Before discussing each step, the data assumed to be available to the analyst is outlined.

[^15]
### 4.1. Required Data

The analyst is assumed to have an estimate of the margin for firm $n$ and data on market outcomes over the same period. In a policy setting, the estimated margin might be calculated directly from internal management accounts - often made available at the request of antitrust authorities. Alternatively, margins might be sourced from published accounts (i.e. reported gross margins, estimated using methods described by De Loecker et al. (2020)).
Data on market outcomes contains information on the products sold in each period, their observed prices and consumer choices. The application in this paper uses household level purchase diary data from Kantar WorldPanel. Aggregate level market data observed at a frequency consistent with price setting behaviour also suffices.

### 4.2. Three-step estimation procedure

Step 1: Price Forecasting I consider two polar empirical models of the relationship between price forecasts and prices set in each period: open and closed-loop.
In an open-loop model, a firm's price setting teams treat prices forecasts as given and do not respond to current prices. Therefore, for firm $n$ in each period $t, \boldsymbol{\Delta}_{n t}^{p}=\mathbf{0}$, and $\boldsymbol{\Upsilon}_{n t}=0$. In contrast, in the closed-loop model the price setting teams have access to the price forecasting model and the time profile of inter-temporal price derivatives are updated as they make decisions. To implement the closed-loop approach, a statistical model is used to approximate the price process.
From an analyst's perspective it is desirable if the approximation to the firms' price forecasting model: (i) can capture the key features of expected high-dimensional promotional price dynamics both across products and over time; (ii) be estimable using sparsely populated historical price data within the policy making time-horizon; and (iii) be straightforward to use to generate the time profile of dynamic price derivatives up to $H$-periods ahead in each period $t$.
One possibility is to use a dynamic factor model (DFM). A DFM provides a flexible, data-driven price forecast model that is easy to implement, is well suited to high-dimensional applications, and can be implemented with missing data. ${ }^{27}$ Under this model each products' price process is approximated as a weighted sum of a low dimensional set of time-varying price factors that follow a $\kappa$-order Markov process. The state space representation of the dynamic factor model of the price process is

$$
\begin{align*}
\boldsymbol{p}_{t} & =\boldsymbol{\Lambda} \boldsymbol{F}_{t}+\boldsymbol{\epsilon}_{t}  \tag{15}\\
\boldsymbol{F}_{t+1} & =\boldsymbol{A} \boldsymbol{F}_{t}+\boldsymbol{U}_{t+1} \tag{16}
\end{align*}
$$

[^16]where $\boldsymbol{F}_{t}=\left[\boldsymbol{f}_{t}, \ldots, \boldsymbol{f}_{t-\kappa}\right]^{\top}$ is a $\kappa R$ vector comprising of a low dimensional set of $R$-vector, $\boldsymbol{f}_{t}$, capturing underlying price trends or 'factors'. Also, $\boldsymbol{\Lambda}=[\boldsymbol{L}, \mathbf{0}, \ldots, \mathbf{0}]$ is a $J \times \kappa R$ matrix and $\boldsymbol{L}=\left[\boldsymbol{\lambda}_{1}^{\top}, \ldots, \boldsymbol{\lambda}_{J}^{\top}\right]^{\top}$ be a $J \times R$ matrix of factor loadings of factor loadings. Standard normalisations on factors and their loadings are imposed. ${ }^{28}$ Finally, $\boldsymbol{\epsilon}_{t}$ is $J$-vector of price shocks and $\boldsymbol{U}_{t+1}=\left[\boldsymbol{u}_{t+1}, 0 \ldots, 0\right]^{\top}$ is a $\kappa R$ vector containing innovations to $\boldsymbol{F}_{t+1}$ where
\[

\left[$$
\begin{array}{c}
\boldsymbol{\epsilon}_{t}  \tag{17}\\
\boldsymbol{u}_{t+1}
\end{array}
$$\right] \sim N\left(\left[$$
\begin{array}{l}
0 \\
0
\end{array}
$$\right],\left[$$
\begin{array}{cc}
\boldsymbol{\Sigma}_{p p} & 0 \\
0 & \boldsymbol{\Sigma}_{f f}
\end{array}
$$\right]\right)
\]

where $\boldsymbol{\Sigma}_{p p}$ and $\boldsymbol{\Sigma}_{f f}$ are the covariance matrices for equations (15) and (16), respectively. For a given $R$, the price factors can be non-parametrically estimated applying a truncated or 'thin' singular value decomposition to standardised price data. The coefficients of the Markov process are estimated by using a $\operatorname{VAR}(\kappa)$ on the estimated factors. ${ }^{29}$
The $h$-periods ahead forecasted price derivatives for a price change in period $t$ are easy to compute functions of its parameters. ${ }^{30}$

$$
\begin{equation*}
\widehat{\boldsymbol{\Delta}}_{t+h}^{p}:=\boldsymbol{\Lambda} \boldsymbol{A}^{h} \boldsymbol{F}_{t} \boldsymbol{F}_{t}^{\top} \boldsymbol{\Lambda}^{\top}\left[\boldsymbol{\Lambda} \boldsymbol{F}_{t} \boldsymbol{F}_{t}^{\top} \boldsymbol{\Lambda}+\boldsymbol{\Sigma}_{p p}\right]^{-1} \tag{18}
\end{equation*}
$$

They are then used to construct and empirical version of the price dynamics term in the first order condition (eq. (14)). To that end, the $h$-step ahead forecasts of product $k$ 's price derivatives are assumed to be uncorrelated with product $k$ 's $h$-step ahead demand forecast.
A1: $\operatorname{cov}\left(\boldsymbol{\Delta}_{h}^{p}, \boldsymbol{q}_{h}\right)=\mathbf{0}$ for $h=1, \ldots, H$
Under assumption A1, the expectation of the present value of revenues due to the temporary price change in period $t$ is product of the expected price forecast derivative and the expected future quantity demanded. ${ }^{31}$ Given $H$ and a discount factor, the empirical analogue to this term is

$$
\begin{equation*}
\widehat{\boldsymbol{\Upsilon}}_{n t}=\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \boldsymbol{\Delta}_{n h}^{p} \boldsymbol{q}_{h}=\boldsymbol{\Gamma}_{n t} \overline{\boldsymbol{q}} \tag{19}
\end{equation*}
$$

where $\boldsymbol{\Gamma}_{n t}:=\sum_{h=t+1}^{H} \delta^{h-t} \widehat{\boldsymbol{\Delta}}_{n h}^{p}$ and $\overline{\boldsymbol{q}}:=\frac{1}{T} \sum_{t=1}^{T} \widehat{\boldsymbol{q}}\left(\boldsymbol{p}_{t}\right)$ is a $J$-vector of average quantity demanded over $T$ periods estimated using the static demand model from step two of the estimation procedure.

[^17]Step 2: Static Demand Estimation A workhorse IO static demand model is estimated in the second step with aim of recovering short-run demand derivatives. The period of input data should match the time interval used in practice by firms when setting prices as closely as possible. For the UK laundry detergent industry, prices are assumed to be set weekly.
Where the analyst has aggregate market data, Berry (1994) and Berry et al. (1995) can be used to estimate conditional, nested, or random coefficient logit demand models. If, as the case in this paper, micro-data is available, then observed household characteristics can be used to estimate a rich nested logit demand function model. Alternatively, if the dimensionality of the choice set is sufficiently low, then the analyst might choose to non-parametrically estimate a static discrete demand model (Compiani (2022)) or estimate an Almost Ideal Demand System (Deaton and Muellbauer (1980)).
As discussed above, price elasticities and diversion ratios computed using the short-run demand derivatives of are biased estimators of their long run counterparts. This is because they exclude the possibility of inter-temporal substitution. However, short-run demand derivatives estimated using the output of the static demand model, $\widehat{\boldsymbol{\Delta}}_{t}^{q}$, are also biased and inconsistent estimates of the contemporaneous demand responses to a temporary price cut. This is because inventory and price expectations are omitted from the static demand model.
In static discrete demand models with a common marginal utility of income for all consumers, the bias from omitting inventory and price expectations scales demand derivatives through its impact on the price coefficient. Therefore the absolute magnitude of the bias is the same for all own and cross-price demand derivatives: $B_{j k}=B_{j^{\prime} k^{\prime}}=B$ for all $j, k, j^{\prime}, k^{\prime} \in \mathcal{J}$ where $B_{j k}:=\left(\widehat{\Delta}_{j k t}^{q}-\Delta_{j k t}^{q}\right) / \Delta_{j k t}^{q}$.
This restriction approximately holds for more flexible static demand models when switchers and brand-loyal consumers have a similar distribution of marginal utility of income. Hendel and Nevo (2006) find that the derivatives computed from a static demand model omitting inventory and price forecasts overstate demand responses: $B>0$. In turn, magnifying their existing bias as estimators of long-run demand derivatives. ${ }^{32}$

Step 3: Demand dynamics Demand dynamics in the first order condition (eq. (7)) are a function of the element-wise product of the short-run demand response to a temporary price cut and dynamic correction ratios, $\boldsymbol{\Psi}_{t}$. Without a dynamic demand model, the analyst cannot simulate consistent estimates of short-run demand responses to a temporary price cut in period $t, \Delta_{t}^{q}$.

[^18]Alternatively, the analyst can replace them with biased estimates from the static demand model estimated in step two, $\widehat{\boldsymbol{\Delta}}_{n t}^{q}=\boldsymbol{\Delta}_{n t}^{q}(1+B)$. The matrix containing the time profile of current and $H$-step ahead total demand derivatives that entering firms' first order conditions (eq. (14)) is

$$
\begin{equation*}
\boldsymbol{\Delta}_{n t}^{q} \odot\left(1+\boldsymbol{\Psi}_{t}\right)=\widehat{\boldsymbol{\Delta}}_{n t}^{q} \odot \boldsymbol{\theta}_{t} \tag{20}
\end{equation*}
$$

where

$$
\begin{equation*}
\boldsymbol{\theta}_{t}:=\frac{1+\boldsymbol{\Psi}_{t}}{1+B} \tag{21}
\end{equation*}
$$

is a conformable matrix of reduced-form parameters, $\boldsymbol{\theta}_{t}$, whose elements are a function of biasscaled dynamic correction ratios. In the final step of the estimation procedure, the outputs from the previous two steps are combined with price data and firm $n$ 's margins estimated from their internal accounts to estimate these reduced form parameters. ${ }^{33}$
Since accounts are costly to compile, financial statistics tend to be produced periodically (i.e. firm-level annual gross margins). The margins calculated from firm $n$ 's internal accounts, $\widehat{\mu}_{n}$, reflect the cumulative effect of the sequence of pricing decisions across multiple products, cost shocks, and corresponding quantity responses spanning several time periods. As a result, they contain information on the effect that demand and price dynamics have on firms' ability to impart market power.
To leverage the information contained in the profit margin estimated directly from internal accounts covering $T$ periods, the expression for the mark-up from the firm's first order condition is summed over all the periods and products it covers. Dividing through by corresponding revenues gives an expression for the margin in terms of a sequence of purchase quantities, estimated short-run demand derivatives, the firm's price forecasts, and $\boldsymbol{\theta}_{n}=\left\{\Omega_{n} \odot \boldsymbol{\theta}_{t}\right\}_{t=1}^{T}$. The resulting expression in the model for the margin earned over $T$ periods by firm $n$ in the model is

$$
\begin{equation*}
\mu_{n}\left(\boldsymbol{\theta}_{n}\right)=-\frac{\sum_{t=1}^{T} \widehat{\boldsymbol{q}}_{n t}^{\top}\left(\widehat{\boldsymbol{\Delta}}_{n t}^{q} \odot \boldsymbol{\theta}_{t}\right)^{-1}\left(\widehat{\boldsymbol{q}}_{t}^{\top}+\widehat{\boldsymbol{\Upsilon}}_{n t}\right)}{\sum_{t=1}^{T} \widehat{\boldsymbol{q}}_{n t}^{\top} \boldsymbol{p}_{t}} \tag{22}
\end{equation*}
$$

where $\widehat{\boldsymbol{q}}\left(\boldsymbol{p}_{t}\right)$ is a $J$-vector of quantity demanded estimated using the static demand model from step two of the estimation procedure and $\widehat{\boldsymbol{\Upsilon}}_{n t}$ is the estimate of the supply dynamics term from step one.
The final step of this procedure solves for elements in $\boldsymbol{\theta}_{n}$ that set the firm $n$ 's margin derived from internal accounts equal to its counterpart from the model, $\widehat{\mu}_{n}=\mu_{n}\left(\boldsymbol{\theta}_{n}\right)$. With a single margin equation covering multiple products' sales, the $T J_{n}^{2}$ non-zero parameters affected by firm $n$ 's pricing decisions are not point-identified.

[^19]
### 4.3. Identification

$\boldsymbol{\theta}_{n}$ For firm $n$ there are $T J_{n}^{2}$ parameters in $\boldsymbol{\theta}_{n}$ to estimate. While the reported margin contains information on how demand and supply dynamics affects market power levied across multiple purchase cycles, only an aggregate measure of bias across the whole period can be recovered. Accepting this practical limitation, assumption A2 restricts the dynamic correction factors to be mean stationary over the accounting period. ${ }^{34}$

## A2: $\boldsymbol{\Psi}=\mathbb{E}\left[\boldsymbol{I T} \boldsymbol{D} \boldsymbol{R}_{t}^{\boldsymbol{\pi}} \oslash \boldsymbol{D} \boldsymbol{R}_{t}^{\boldsymbol{\pi}}\right] \forall t=1, \ldots, T$

Under this assumption A2, the number of parameters in $\boldsymbol{\theta}_{n}$ is reduced by a factor of $T$. For a single product firm, under A2, $\widehat{\mu}_{n}=\mu_{n}\left(\theta_{n}\right)$ point-identifies $\theta_{n}$. However, for firms with even a handful of products the elements of $\boldsymbol{\theta}_{n}$ are not point identified. Moreover, given that storable good firms often sell tens of products, the bound on $J_{n}^{2}$ parameters is likely to be too wide to be useful in a policy setting. In practice, for use in a policy setting further restrictions are needed. From the model above and the existing literature, the direction and magnitude of bias using a static demand model is expected to differ by promotion status. If the good is promoted, static demand responses overstate total volume responses. For substitutes to the promoted good, total demand responses are understated by the output of static demand models. In line with theoretical and empirical findings, two different aggregate measures of bias adjustment are retained in assumption A3. One for own-price demand derivatives and another for cross-price demand derivatives.
For promoted products, the diagonal elements of $\boldsymbol{\theta}_{n}$ contributing to firm $n$ 's margin are constrained to be the same. This constrains the fraction of profits earned on the promoted good during the sale period that are pulled forward from its future sales to be equal for all firm n's products. Similarly, all non-promoted products are assumed to have the same fraction of current profits lost over future periods to the promoted good in the sale period. This constrains all off-diagonal elements in the non-zero rows of $\boldsymbol{\theta}_{n}$ to be equal. ${ }^{35}$ Asymmetric overall substitution patterns between products is inherited from the static demand estimation.
A3: $\theta_{j j}=\theta_{n}^{\text {own }} \in(0,1]$ and $\theta_{j k}=\theta_{n}^{\text {cross }} \geq 1$ for all $j \neq k, j \in \mathcal{J}_{n}, k \in \mathcal{J}$
Under assumptions A2 and A3, the number of parameters in $\boldsymbol{\theta}_{n}$ reduces from $T J_{n}^{2}$ to two parameters for each firm $n$. The set of values for firm $n$ are defined by the set

$$
\begin{equation*}
\Theta_{n}^{\star}:=\left\{\left(\theta_{n}^{\text {own }}, \theta_{n}^{\text {cross }}\right) \in\left[\underline{\theta}_{n}^{\text {own }}, \bar{\theta}_{n}^{\text {own }}\right] \times\left[1, \bar{\theta}_{n}^{\text {cross }}\right] \mid \widehat{\mu}_{n}=\mu_{n}\left(\theta_{n}^{\text {own }}, \theta_{n}^{\text {cross }}\right)\right\} \tag{23}
\end{equation*}
$$

where $\underline{\theta}_{n}^{\text {own }}$ and $\bar{\theta}_{n}^{\text {own }}$ are the minimum and maximum values of $\theta_{n}^{\text {own }}$ when $\theta_{n}^{\text {cross }}$.
The upper bound, $\bar{\theta}_{n}^{\text {cross }}$, is application specific and chosen by the policy analyst. Plausible values $\bar{\theta}_{n}^{\text {cross }}$ can be obtained by making assumptions on the nature of the decline in

[^20]inter-temporal substitution over time. For example, in line with Erdem et al's (2003) simulated demand responses to a temporary price cut of a leading US Ketchup brand, assuming inter-temporal substitution declines exponentially over time and that almost all substitution is completed by the mean inter-purchase duration.
Having chosen $\bar{\theta}_{n}^{\text {cross }}$, the analyst can fix a grid of values for $\theta_{n}^{\text {cross }} \in\left[1, \bar{\theta}_{n}^{\text {cross }}\right]$ and use rootfinding algorithm to find the value $\theta_{n}^{\text {own }}$ given $\theta_{n}^{\text {cross }}$ that sets the margin estimated from internal accounts equal to its counterpart in the model.

Bounds on $\boldsymbol{D} \boldsymbol{D} \boldsymbol{R}_{\boldsymbol{j k}}^{\boldsymbol{\pi}}$ Under assumptions A2 and A3, the elements of $\Theta_{n}^{\star}$ can be used to compute

$$
\begin{equation*}
\frac{1+\Psi_{j k}}{1+\Psi_{j j}}=\frac{\theta_{n}^{\text {cross }}}{\theta_{n}^{\text {own }}} \tag{24}
\end{equation*}
$$

for $j \neq k$ where $j \in \mathcal{J}_{n}$ and $k \in \mathcal{J}$. In turn, this facilitates consistent estimation of the bounds on dynamic diverted value ratios, $D D R_{j k}^{\pi}$. Specifically, the bounds are given by

$$
\begin{align*}
& \underline{D D R}_{j k}^{\pi}:=\min \left\{\frac{\theta_{n}^{\text {cross }}}{\theta_{n}^{\text {own }}} \text { s.t. }\left(\theta_{n}^{\text {own }}, \theta_{n}^{\text {cross }}\right) \in \Theta_{n}^{\star}\right\} D R_{j k}^{\pi}  \tag{25}\\
& \overline{D D R}_{j k}^{\pi}:=\max \left\{\frac{\theta_{n}^{\text {cross }}}{\theta_{n}^{\text {own }}} \text { s.t. }\left(\theta_{n}^{\text {own }}, \theta_{n}^{\text {cross }}\right) \in \Theta_{n}^{\star}\right\} D R_{j k}^{\pi} \tag{26}
\end{align*}
$$

where the short-run diverted value ratios is calculated using the static demand model's outputs, margins estimated from firms' accounts, and observed prices over $T$ periods. ${ }^{36}$

## 5. Results

The remainder of the paper applies this empirical methodology to the UK laundry detergent industry. The sets $\left(\theta_{n}^{\text {own }}, \theta_{n}^{\text {cross }}\right) \in \Theta_{n}^{\star}$ are estimated for firm A and firm $B$ in each year from 2002 to 2012 and the effect of increased product compaction over the sample is analysed.

### 5.1. Step 1: Price forecasting model

The baseline version of the estimated model, the high-dimensional dynamic factor model (eq. (15) and (16)) described in section 5.1 is used. ${ }^{37}$ It is estimated using average weekly prices from purchases made by households in the Kantar Worldpanel data from over 11 years.

[^21]Table 3: Price dynamics: breakpoints, $R^{\star}$, and $\kappa^{\star}$

| Regime | Start |  | End |  | Weeks | $R^{\star}$ | $\kappa^{\star}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Year | Week | Year | Week |  |  |  |
| 1 | 2002 | 1 | 2003 | 29 | 81 | 3 | 2 |
| 2 | 2003 | 30 | 2006 | 45 | 172 | 2 | 1 |
| 3 | 2006 | 46 | 2008 | 24 | 83 | 8 | 1 |
| 4 | 2008 | 25 | 2009 | 24 | 52 | 5 | 1 |
| 5 | 2009 | 25 | 2010 | 41 | 69 | 2 | 1 |
| 6 | 2010 | 42 | 2011 | 42 | 53 | 7 | 1 |
| 7 | 2011 | 43 | 2012 | 42 | 52 | 2 | 1 |

Given this long horizon, structural breaks might occur in the underlying price process. The iterative procedure described in Baltagi et al. (2021) is used to detect the presence, number, and location of structural breaks. This includes the choice of its hyper-parameters and necessary minor modifications to enable calculation of its critical values as the degrees of freedom increase. ${ }^{38}$
A summary of the detailed test results above are shown in Table 3. The first four columns list the regimes, the start and end weeks, and the their length in weeks. The last two columns show the number of latent factors, $R^{\star}$, and lags of the factor VAR, $\kappa^{\star}$, in each regime. The number of factors in each regime uses penalised least squares (Bai and $\operatorname{Ng}(2002)$ ). The Bayes Information Criteria is used to select and number lags, $\kappa^{\star}$, in the factor VAR.
This procedure detects six structural breaks in the statistical model of price dynamics over the sample period. The first three regimes cover two-thirds of the sample - each lasting between 1.5 and 3 years. In contrast, each of the last five years contains a structural break. The changing price regimes in the latter half of the sample coincides with the introduction of new super concentrated and gel formats and subsequent acceleration in the rollout of increasingly compacted products across the set of products firms sold. ${ }^{39}$ In turn, suggesting that continued evolution of the set of products being sold contributed to periodic re-optimisation of pricing strategies.

[^22]
### 5.2. Step 2: Static demand model

Given the policy focus of this paper, a demand model often employed in antitrust investigations - the nested logit model - is estimated. The nested logit model is estimated using the purchase diary data from Kantar Worldpanel. The availability of Kantar WorldPanel micro-data enables the use of observed household characteristics to estimate a rich demand model.
Household $i$ elects to purchase good $j$ from a market $t$ to maximise conditional indirect utility

$$
\begin{equation*}
U_{i j t}=\boldsymbol{X}_{j}^{\top} \boldsymbol{\beta}_{i}+\alpha_{i} p_{j t}+\xi_{j t}+\varepsilon_{i j t} \tag{27}
\end{equation*}
$$

where $\boldsymbol{X}_{j}$ is a $K$-vector of observed product attributes. The outside good is denoted by $j=0$ and represents the decision not to purchase in market $t$.
Because the model is estimated using consumer micro-data, the parameters $\alpha_{i}$ and $\boldsymbol{\beta}_{i}$ depend on observed household characteristics. There are two components of utility not observed by the analyst: $\xi_{j t}$ is an unobserved product-market component of utility common to all consumers and $\varepsilon_{i j t}$ is a private household-specific utility shock. The unobserved product-market components are observed by all firms prior to setting prices. As a result, prices are likely to be correlated with $\xi_{j t}$ and are not independent from the unobserved component of demand.

Estimation The demand model is estimated using a random sample of five purchases in each week from the Kantar Worldpanel purchase diary data resulting in 2,810 choice occasions. The number of products in choice sets ranges from 87 to 187 with a median of 120 products. The set of products sold in each week is partitioned into four nests based on the number of washes contained in each product: small (S), medium (M), large (L), and extra large (XL). The size boundaries of these groups correspond to the 25th, 50th, and 75 th quantile of distribution of washes in each calendar year. ${ }^{40}$
In the UK laundry detergent Kantar WorldPanel data 12 percent of product-week market shares are zero. As a result, it is not possible to use Berry (1994) to deal with price endogeneity. Therefore, the control function approach proposed by Petrin and Train (2010) is used. ${ }^{41}$

Identification The static demand model parameters, $\alpha, \beta$ and $\rho$ are common to all consumers. They are identified by variation in prices, product characteristics and fitted control function residuals within consumers' choice sets. The parameters that interact consumer with price or product characteristics are identified from variation in choices across different types of consumers. The nesting parameter, $\lambda$, is identified by variations in the ex-ante expected utility of purchasing a product from a given nest for each consumer.

[^23]Table 4: Choice models: parameter estimates

|  | Nested Logit |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Variables | $\theta$ | se | $\theta$ | se |
| Price Params |  |  |  |  |
| Price | -0.275 | 0.021 | -0.581 | 0.136 |
| Price x Income | 0.179 | 0.039 | 0.184 | 0.029 |
| Characteristics |  |  |  |  |
| Washes | 0.118 | 0.163 | 4.484 | 1.949 |
| Washes per Eq. Ad. | -0.337 | 0.095 | -0.371 | 0.771 |
| Dosage | -1.222 | 0.108 | -1.058 | 0.284 |
| Dosage x Liquids | 0.279 | 0.067 | 0.575 | 0.381 |
| Other Params |  |  |  |  |
| $\lambda$ | 0.526 | 0.035 | 0.551 | 0.049 |
| $\rho$ |  |  | 0.297 | 0.126 |
| Detergent Fixed Effects | Yes |  | Yes |  |
| N | 2,810 |  | 2,810 |  |
| Log-Likelihood | $-12,203$ |  | $-12,199$ |  |

Results The results of the nested logit estimation are displayed in Table 4. It contains the parameters and standard errors of the nested logit model with and without the control function. ${ }^{42}$
The specification of conditional indirect utility include interactions between price and a proxy for household income. ${ }^{43}$ Product characteristics include the size of the product purchased and the dosage - the amount of material (recommended) for use in a single wash. To control for household size, I also include the amount of washes purchased per equivalent adult in the household. ${ }^{44}$ Detergent fixed effects are also included.
In both nested logit models the price coefficient is negative and households with higher income have a lower marginal utility of income. When the control function is included, the price coefficient (scaled by the nesting parameter) approximately doubles from -0.523 to -1.054.
This is consistent with the correlation between unobserved product demand factors being positive. It is also reflected by the control function parameter, $\rho$, being positive and statistically significantly different from zero.
Table 4 also shows that households positively value larger pack-sizes. However, for smaller households this is less pronounced. This is consistent with smaller households living in smaller accommodation and facing a higher cost of storage.

[^24]The amount of material needed to do a single wash (i.e. dosage) is negatively valued, especially for 'solid' detergents. This is also consistent with the fact that households value storage space. When the dosage is lower, households can store more washes without necessarily occupying more storage space. Indeed, this is one of the driving factors behind the success of the new super-concentrated and gel detergent products. By itself, this suggests that the presence of inter-temporal demand links through inventories is, as expected, a source of mis-specification for this static demand model.
The nesting parameter, $\lambda$, is 0.551 and is statistically significantly different from 1 . This indicates that that there are some unobserved correlations in the utility between detergents of similar sizes and rejects the independence of irrelevant alternatives imposed by a conditional logit.

### 5.3. Step 3: Estimating $\left(\boldsymbol{\theta}_{n}^{\text {own }}, \boldsymbol{\theta}_{n}^{\text {cross }}\right) \in \Theta_{n}^{\star}$

As is often the case in policy analysis, having reviewed the available evidence there a number of inputs chosen by the analyst. In this case, these include modifications to reflect key idiosyncratic industry features and parameter calibrations. Specifically, to estimate $\boldsymbol{\theta}_{n}$ the analyst specifies the optimisation window $(H)$, the firm's discount factor, the type of price forecast model, and $\bar{\theta}_{n}^{\text {cross }}$ for the baseline model. Sensitivity analysis evaluates the robustness of conclusions drawn to input assumptions. Next, I describe these choices for a baseline model of the UK laundry detergent industry over the sample period.

Baseline Model In the baseline model, the optimisation horizon is set equal to the median inter-purchase duration observed in the data in each year (see Table 2). Unlike the median, the mean is affected by a handful of households with long gaps between purchases - to whom price dynamics are unlikely to be a first order purchase determinant. For this reason, an analysis with $H$ equal to mean inter-purchase duration is retained as a scenario to evaluate the sensitivity of the baseline model with a longer optimisation window.
The firm's weekly discount factor reflects the cost of capital obtainable at the time from financial markets in the sample. For this purpose, the sample is split pre and post the 2008 financial crisis. Reflecting underlying movement in UK base rates, the firm's weekly discount factor is $\delta=0.998$ for the period 2002 to 2008 and $\delta=0.999$ thereafter.
In the baseline model, prices are forecast using the DFM estimated in the first step of the estimation procedure. The open loop forecasting model, in which $\boldsymbol{\Gamma}_{n t} \overline{\boldsymbol{q}}=0$, is retained as another scenario to evaluate the robustness of the key conclusions to the closed-loop assumption in the baseline model.
$\bar{\theta}_{n}^{\text {cross }}$ is chosen assuming that most inter-temporal substitution are purchases that would have occurred in the near future. Specifically, the fraction of switchers is assumed to exponentially

Table 5: $\bar{\theta}_{n}^{\text {cross }}$ by optimisation horizon

|  | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $H=$ median | 1.36 | 1.59 | 1.59 | 1.59 | 1.59 | 1.59 | 1.59 | 1.82 | 1.82 | 1.82 | 1.82 |
| $H=$ mean | 1.83 | 2.06 | 2.30 | 2.30 | 2.30 | 2.30 | 2.55 | 2.80 | 2.80 | 3.04 | 3.04 |

decline over the chosen optimisation window. ${ }^{45}$

$$
\begin{equation*}
\bar{\theta}_{n}^{\text {cross }}=\sum_{h=0}^{H} \delta^{h} \exp (-r h) \tag{28}
\end{equation*}
$$

Table 5 shows the values for $\bar{\theta}_{n}^{\text {cross }}$ under the mean and median inter-purchase duration. In the baseline model, the parameter controlling the rate of exponential decline, $r$, is calibrated so that inter-temporal substitution is largely complete by period $H$. It is assumed that only two percent of contemporaneous switchers would have purchased a rival product in the $H$-th period after the promotion.
The resulting values for $\bar{\theta}_{n}^{\text {cross }}$ are 1.36 in 2002, 1.59 between 2003 to 2008, and 1.84 from 2009 onwards. Setting the optimisation window to the mean, rather than the median, inter-purchase duration provides higher values for $\bar{\theta}_{n}^{\text {cross }}$. The sensitivity of the baseline model to $\bar{\theta}_{n}^{\text {cross }}$ is part of the scenario with a longer optimisation window. ${ }^{46}$
Using inter-purchase duration to compute $\bar{\theta}_{n}^{\text {cross }}$ and the forecast horizon links the estimates of $\left(\theta_{n}^{\text {own }}, \theta_{n}^{\text {cross }}\right)$ to the changes in the size of products sold. With more washes in inventory, interpurchase durations increase and inter-temporal substitution is a larger component of overall substitution. All else equal, omitting demand dynamics might be expected to increase bounds on $\theta_{n}^{\text {own }}$ and $\theta_{n}^{\text {cross }}$ when products contain more washes.
Finally, laundry detergents are subject to VAT in the UK. The first order conditions are updated accordingly. For simplicity, changes in VAT rates are assumed not to be anticipated by firms and computations reflect the VAT prevailing in the UK at the time. ${ }^{47}$
The results of the subsequent analysis under the different scenarios noted above are presented in Appendix C. This sensitivity analysis shows that while some magnitudes change, the key conclusions are qualitatively unchanged.

[^25]Estimation In the final step of the estimation procedure, the baseline model is combined with market outcome data and estimated margins to find values of $\boldsymbol{\theta}_{n}$ that solve equation (22) for firms A and B in each year from 2002 to 2012.
In the context of antitrust investigations, brand level margins may be available over several years. However, in my case, I only have access to published annual accounts. Therefore, the published gross margins in firm A's and firm B's annual accounts from 2002 to 2012 are assumed to be good approximations to the economic margins earned on sales of their laundry detergent portfolio in the UK. ${ }^{48}$
To compute the elements of $\Theta_{n}^{\star}$, fix a grid of $L$ values for $\theta_{n}^{\text {cross }}$ spanning 1 and $\bar{\theta}_{n}^{\text {cross }}$. Next, for each element of $\theta_{n}^{\text {cross }}$ in the grid, plug in observed prices, quantities, expected price dynamics and other parameterised components into the model's expression for the margin, $\mu_{n}\left(\theta_{n}^{\text {own }} \mid \theta_{n}^{\text {cross }}\right)$. Then solve for $\theta_{n}^{\text {own }}$ so that $\widehat{\mu}_{n}=\mu_{n}\left(\theta_{n}^{\text {own }} \mid \theta_{n}^{\text {cross }}\right)$. Interpolating the results defines the set of solution pairs: $\left(\theta_{n}^{\text {own }}, \theta_{n}^{\text {cross }}\right) \in \Theta_{n}^{\star} .{ }^{49}$

Results When $\theta_{n}^{\text {own }}=\theta_{n}^{\text {cross }}=1$ the static demand model dictates substitution patterns and the model's margin calculation over $T$ periods contains no adjustment for demand dynamics. If, as anticipated, the static demand model overstates own price elasticities and understates cross price elasticities, the model generates too little market power and cannot match observed margins.
Figure 1 shows $\left(\theta_{n}^{\text {own }}, \theta_{n}^{\text {cross }}\right) \in \Theta_{n}^{\star}$ in each year for firm A (the left panel) and for firm B (the right panel). They trace out the combination of asymmetric inter-temporal substitution adjustments for the promoted and rival products that equate the model's margin to that estimated from accounting data. In line with the findings in dynamic demand literature, as expected, $\left(\theta_{n}^{\text {own }}, \theta_{n}^{\text {cross }}\right) \in \Theta_{n}^{\star}$ are positively correlated.
This correlation arises because both own and cross product inter-temporal substitution increase market power. The strongest inter-temporal substitution for the promoted good, $\theta_{n}^{\text {own }}=\underline{\theta}_{n}^{\text {own }}$, occurs when there are no demand dynamics with rival products. However, as $\theta_{n}^{\text {cross }}$ increases and approaches its upper bound, the firm accrues market power from recaptured future sales of its substitute products. As a result, the model can match the margin when future sales of the promoted good are perceived by consumers as more distant substitutes for current sales.
The size of the adjustment and evolution over time differs for firms A and B. Between 2002 and 2006, 20 to 30 percent of firm $A$ 's of its uplift in profits from the promotion is drawn from its future profits. ${ }^{50}$ From 2007 onwards, the bias due to the omission of inter-temporal substitution increases. By 2011, a larger adjustment to the static demand model's derivatives is required

[^26]to match margins. In 2011 and 2012, over 50 percent of $A$ 's promotional profits draw down on the present value of its promotional products' future sales.
The bias due to the omission of inter-temporal substitution increases because products contain more consumption units following storage product compaction innovations in 2007 and 2008 (see Figure 3 in Appendix A). Because firm A hold the purchased price per wash broadly constant (see Figure 4 in Appendix A), increasing the number of washes leads to proportionately higher prices. In the static nested logit demand model estimated, higher prices lead to more elastic demand responses and therefore even lower margins in the unadjusted supply model. This effect is compounded by the fact that product innovation allows firm A to reconfigure their portfolio to contain more distant substitutes over time. ${ }^{51}$
The right panel of Figure 1 shows the sets $\left(\theta_{B}^{\text {own }}, \theta_{B}^{\text {cross }}\right) \in \Theta_{B}^{\star}$ are less dispersed over time than for firm A. From 2002 to 2008, between 31 and 39 percent of firm $B^{\prime} s$ promotional profits are drawn from its promoted products' future sales.
After 2008-as product compaction is applied to more of its products - these figures increase to between 39 and 45 percent in 2009 and 2010. However, after 2011 only 31 to 35 percent of the promotional profit uplift are accelerated future purchases. At first glance this is puzzling - like firm A, firm B's compaction innovation also led to increased product sizes and an increasingly different product range. ${ }^{52}$ However, this can be explained by differences in pricing strategy and an observed reduction in market power after 2010.
Unlike firm A, Figure 4 shows that firm B chose to hold posted price per wash stable, but offer increasingly deep discounts from 2007 onwards. This leads to a lower purchased price per wash. Even though product sizes increase after 2007 as product are compacted, firm B's prices increase to a lesser degree than firm A and the demand response is only slightly more elastic. This mitigates the need for increasingly large adjustments to match observed margin from 2008 to 2010.
The subsequent increase bounds on $\theta_{B}^{\text {own }}$ can be attributed to the combination of a 10 percentage point reduction in the observed margin in 2011 and 2012 and increasingly deep discounting. Together, the effect they have on market power are large enough to more than offset the increased elasticity due to continued growth in product prices.

## 6. Dynamic GUPPI

The 2010 US Horizontal Merger Guidelines introduced upwards price pressure measures: UPP (Farrell and Shapiro (2010)) and the gross upwards price pressure index (GUPPI) (Moresi (2010)). Price pressure tests have been used to screen mergers and assess closeness of

[^27]Figure 1: $\left(\theta_{n}^{\text {own }}, \theta_{n}^{\text {cross }}\right)$ for Firm A and Firm B in each year from 2002 to 2012

competition - particularly for differentiated products. ${ }^{53}$
The GUPPI is a measure of the strength of the closeness of substitution between two products, $j$ and $k$. It can be expressed using volume or profit diversion ratios. Using the volume measure, the diverted sales from product $j$ to $k$ multiplies the margin on product $k, \mu_{k}$, and the ratio of the two prices. Alternatively, GUPPI is the margin earned on sales of product $j, \mu_{j}$, multiplied by the diverted value ratio from product $j$ to $k$.

$$
\begin{equation*}
G U P P I_{j k}:=D R_{j k} \mu_{k} \frac{p_{k}}{p_{j}}=D R_{j k}^{\pi} \mu_{j} \tag{29}
\end{equation*}
$$

To calculate GUPPI in a policy setting requires estimates of diversion ratios, margins and prices. In practice, the market power is proxied by margins measured or derived from a firm's (internal) accounting records. ${ }^{54}$ As such, it is typically measured over a period of time (i.e. one year). Therefore when demand and supply dynamics are present, the margin implicitly reflects both immediate and inter-temporal substitution.

[^28]Where data permits, diversion ratios are estimated using the output of a demand model. As noted above, for storable goods the volume and profit diversion ratios are downward biased when inter-temporal substitution is omitted from the procedure used to estimate demand. ${ }^{55}$ In this case, the diversion ratios are biased and are not consistent with the competitive dynamics that affect firms' market power. To remedy this a dynamic diverted value ratio is required.
Replacing the biased static diverted value ratios in the GUPPI formula (equation (29)) with its long-run counterpart defines a dynamic generalised upward pricing pressure index, dGUPPI. ${ }^{56}$

$$
\begin{equation*}
d G U P P I_{j k}:=D D R_{j k}^{\pi} \mu_{j} \tag{30}
\end{equation*}
$$

Estimation Estimates of $\left(\theta_{n}^{\text {own }}, \theta_{n}^{\text {cross }}\right) \in \Theta_{n}^{\star}$ can be used to construct upper and lower bounds on $D D R_{j k}^{\pi}$ can be then plugged into equation (30) to compute bounds of a dynamic version of the GUPPI test, $d G U P P I_{j k}$.

$$
\begin{equation*}
d G U P P I_{j k} \in\left[\underline{D D R}_{j k}^{\pi}, \overline{D D R}_{j k}^{\pi}\right] \widehat{\mu}_{j} \tag{31}
\end{equation*}
$$

The main drawback is that point-identification of dGUPPI is lost. However, as shown in the policy application below, dGUPPI being set-valued does not necessarily preclude its efficacy in a policy setting.
Given that estimation of a sufficiently flexible dynamic demand model is unlikely to be feasible within the policy making time horizon, and simulation of the post-merger outcomes made even more complex by the presence of multiple equilibria, it is hoped this new policy tool will be useful for antitrust analysis of storable good industries.

Hypothetical merger To illustrate how the estimates of $\left(\theta_{n}^{\text {own }}, \theta_{n}^{\text {cross }}\right) \in \Theta_{n}^{\star}$ can be used, suppose brand A and brand E are produced by different firms. Further assume that the firm producing A is buying brand E from the rival firm at the end of 2012. For the purpose of this exercise assume brand A is produced by firm A and brand E by firm $\mathrm{B} .{ }^{57}$
Further, for the purpose of this hypothetical acquisition assume that brand A is the closest substitute for brand E. Therefore, if empirical policy analysis can show that brands A and E are sufficiently distant competitors, then the acquisition is more likely to be permitted.
To assess whether the proposed acquisition is likely to lead to a significant lessening of competition, the analyst can calculate dGUPPI. Allowing for some tolerance in potential efficiencies arising through joint production, it is assumed that the policy analyst views a

[^29]dGUPPI below five percent as sufficiently low to allow the brand merger to occur without giving rise to anti-competitive concerns. ${ }^{58}$
The dGUPPI is calculated in each year to gather evidence on the likelihood of prices rising by examining the strength of the incentive to increase the prices in previous years. Two dGUPPI indices are computed; one for the price increase of brand $A$ and another for the price increase of brand E. In both cases they are compared to a static GUPPI.
The static GUPPI for brand A and brand E is shown by the red lines in the left and right panels of Figure 2, respectively. Between 2002 and 2007 the $G U P P I_{A E}$ is relatively stable and averages 6.4 percent. After 2008, it first drops below the five percent threshold: starting from 6.1 percent in 2007 it declines to 3.8 percent in 2012 . This drop in the $G U P P I_{A E}$ is driven by a decline in the diverted value ratio - it fell from 11.9 percent in 2007 to 7.6 percent in 2012. Detergent level analysis indicates that drop occurs because brand A became a leading brand for the new gel formats rolled out from 2008 onwards. Prior to that brand E's powder and tablets were perceived to be closer substitutes by brand A customers.
$G U P P I_{E A}$ is below the five percent threshold in each year over the period - an average of 4.2 percent across the period. Therefore, relying solely on a static GUPPI measures, an analyst might conclude that although the brands were closer competitors in the past, innovation in product formats and product compaction since $2007 / 8$ led to the brands being sufficiently distant competitors by 2012 to permit the acquisition for both brand A's and brand E's customers.
However, once the diversion ratios are adjusted for bias due to the omission of demand dynamics, the analyst's conclusions are reversed. The resulting range of possible values of the dGUPPI is shown by the blue bands in Figure 2.
The left panel shows the range of possible values for $d G U P P I_{A E}$. It shows that once intertemporal substitution is accounted for, the price increase predicted by $d G U P P I_{A E}$ in 2012 lies between 8.4 and 14.3 percent - well above the five percent threshold. By itself, this finding would be sufficient to reverse the conclusions of the static GUPPI analysis.
The right panel shows the range of possible values for $d G U P P I_{E A}$. By 2012, the range of possible values for $d G U P P I_{E A}$ is 6.2 to 10.9 percent. Unlike the static GUPPI, the analyst now cannot rule out that the dGUPPI lies strictly below the five percent threshold in all years. These findings would support a prohibition of the firm producing brand A to acquire brand E.

[^30]Figure 2: GUPPI and dGUPPI: brand A and brand E, 2002 to 2012



## 7. Conclusion

This paper develops a new approach to estimate bounds on sets of dynamic diverted value ratios for storable goods industries. To identify substitution dynamics missing from static demand models, the approach leverages information on market power over multiple purchase cycles contained in margins derived from a firm's internal accounts. To estimate the bounds, these are equated to the margin from a dynamic demand and supply model for forward-looking storable good firms.
By combining the output of a price forecasting and static demand models with prices and margins, I recover parameters that capture the effect of inter-temporal substitution omitted from static models of firms' pricing incentives. These parameters are used to construct bounds on estimates of dynamic diverted value ratios.
I show dynamic diverted value ratios can be combined with margins in a new generalised upward price pressure index adjusted for demand dynamics - dGUPPI. This new empirical policy tool extends the existing GUPPI test and can be used to evaluate mergers in industries with dynamic demand.
To illustrate this new framework, I apply it to the UK laundry detergent industry from 2002 to 2012. I show that static diversion ratios are severely downward biased measures of their longrun counterparts. As expected, this bias grows when inter-temporal substitution becomes an
increasingly large component of aggregate consumer switching in response to per unit savings in storage costs and intensified promotional activity.
To show how the set-valued estimates of policy inputs can be used to aid policy-making I show how sets of dynamic diverted value ratios produce dGUPPI ranges that can help a competition authority evaluate the likelihood of unilateral effects arising from a hypothetical brand acquisition from a rival. In turn, I show that the use of set-valued policy inputs is central to avoiding policy errors.
The application demonstrates that this new approach is well suited to policy work. It only requires standard empirical methods and readily available data on margins and market-level industry outcomes. As a result, it can be easily implemented within the policy making timehorizon. More generally, it highlights the potential of empirical analysis producing set-valued outcomes to aid the policy-making process.

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## Appendices

## A. Product compaction in the UK laundry detergent industry

Following a series of industry-wide initiatives over 2002 to 2005 that sought to reduce the environmental impact, dosage per wash was gradually decline. However, this sped up dramatically following the introduction of two new formats - super concentrated liquid and gel - in 2007 and 2008, respectively.
From 2007 onwards product compaction intensified. The top panel of Figure 3 shows that new product formats quickly capture market share over 2007-2010 and quickly replaced similar but more bulky alternatives (i.e. tablet and non-concentrated liquids). After both new format introductions, efforts to gradually reduce other formats dosage to improve their storage spacecompaction tradeoff continued. Over the period 2008 to 2012, the average dose per wash fell by 15 percent for powder, 9 percent for capsules and 7 percent for tablets.
As dosages have declined over time, so has the physical storage space occupied by the material needed per wash. In response, firms gradually produce larger pack sizes over time. ${ }^{59}$ The average number of washes per purchased pack have increased by around 50 percent from 17 washes in 2002 to 26 washes in $2012 .{ }^{60}$
To explore role that innovation through new products and compaction had on consumer demand on pricing, Figure 3 shows the share of revenue by format and the number of washes in a pack in each year over the sample period.
The top panel shows that new product formats quickly capture market share over 2007-2010 and quickly replaced similar but more bulky alternatives (i.e. tablet and non-concentrated liquids). After both new formats' introductions, efforts to gradually reduce other formats dosage to improve their storage space-compaction tradeoff continued. Over the period 2008 to 2012, the average dose per wash fell by 15 percent for powder, 9 percent for capsules and 7 percent for tablets.
The impact of new formats and smaller dosage for existing formats on the composition on product sizes is shown in the bottom panel. As expected, this storage cost savings per wash appear to have contributed to the increasing popularity of larger packs of laundry detergent. In 2002, around $75 \%$ of household spend was on SKUs with fewer than 24 washes, 10 years later

[^31]Figure 3: Revenue share by pack size from 2002 to 2012


this figure was less than 35 percent. Notably, as the product compaction is sped up across the firms' range of products, consumers increasingly shift to larger products.
The effect of reduced storage costs and larger pack sizes enabled household to service consumption needs from inventory for longer. The result was increased household's inter-purchase durations (see Table 2) in Section 2. Faced with fewer consumer interactions, firms appear to compete more intensely to attract consumers by engaging in deeper and more frequent discounting.
Figure 4 uses a series of box plots to display the distribution of price per wash in each quarter from 2002 to 2012. The red line plots the average posted price per wash and the green line plots the average price per wash of purchased products. The top panel shows the price per wash distribution for firm A and bottom panel shows this distribution for firm B.
For both firm A and firm B, the whiskers and inter-quartile range of box plots from first quarter in 2002 up to the final quarter in 2006 are relatively constant. Over this period, the whiskers tend to lie between 10 p and 30 p per wash and the interquartile ranges lie between 15 p and 23 p per wash. From the first quarter in 2007 onwards, the whiskers and inter-quartile ranges of the box plot fan out for both firm A and firm B. This increased price dispersion coincides with the introduction of new product formats in 2008 for firm A and 2007 for firm B.
Following the success of the new formats, a larger number of products sold are compacted further over 2009 to 2012. In turn, products sold contain more washes, inter-purchase durations continue to increase and there is a concomitant increase in the frequency and depth of discounting.
Other contributing explanations for the observed changes in pricing behaviour towards the end of sample period, inter alia, include increased price sensitivity of households due to changes in the macroeconomic climate and changes in pricing strategy resulting from firm investments sophistication of pricing tools (i.e. due to increased computational power and improved ability to leverage larger data sets).

## B. Dynamic Storable Good Model

## B.1. First Order Conditions

First consider the case of two single product firms competing in a duopoly setting prices to maximise expected discounted profits over only two periods. In this case, for firm $n$ selling only product $j$ the discounted profits earned over the periods are given by

$$
\pi_{n t}^{N P V}=\left(p_{j t}-m c_{j t}\right) q_{j}\left(\boldsymbol{p}_{t}, \boldsymbol{\omega}_{t}\right)+\delta \mathbb{E}_{t}\left(p_{j, t+1}-c_{j, t+1}\right) q_{j}\left(\boldsymbol{p}_{t+1}, \boldsymbol{\omega}_{t+1}\right)
$$

where $m c_{j t}$ is the marginal cost of production for product $j$ in period $t$.
Firm $n$ 's optimal price of product $j$ in the current time period satisfies the first order condition given below

Figure 4: Price per wash (PPW) distribution: firms A and B from 2002 to 2012.


$$
\begin{align*}
q_{j t}+\frac{\partial q_{j t}}{\partial p_{j t}} m_{j t}+\delta \mathbb{E}\left[\left(\frac{\partial q_{j, t+1}}{\partial p_{j, t+1}} \frac{\partial p_{j, t+1}}{\partial p_{j t}}+\frac{\partial q_{j, t+1}}{\partial p_{k, t+1}} \frac{\partial p_{k, t+1}}{\partial p_{j t}}\right) m_{j, t+1}+\frac{\partial p_{j, t+1}}{\partial p_{j t}} q_{j, t+1}\right] & =0  \tag{32}\\
\Longrightarrow q_{j t}+\frac{\partial q_{j t}}{\partial p_{j t}} m_{j t}+\delta \mathbb{E}\left[\frac{d q_{j, t+1}}{d p_{j t}} m_{j, t+1}+\frac{d p_{j, t+1}}{d p_{j t}} q_{j, t+1}\right] & =0  \tag{33}\\
\Longrightarrow q_{j t}+\frac{\partial q_{j t}}{\partial p_{j t}} m_{j t}+\delta \mathbb{E} \frac{d q_{j, t+1}}{d p_{j t}} m_{j, t+1}+\delta \mathbb{E}_{t} \frac{d p_{j, t+1}}{d p_{j t}} q_{j, t+1} & =0 \tag{34}
\end{align*}
$$

where $m_{j t}:=p_{j t}-m c_{j t}$ and product $k$ is produced by the rival firm.
The first two terms in equation (32) are the usual terms in the first order conditions for firms competing in static differentiated Bertrand duopoly game. The third term in equation (32) shows how a change in current prices affects the expected discounted profits in the second period through two channels.
First, a change in the price of product $j$ in period $t$ affects quantity demanded of product $j$ in period $t+1$ indirectly through the effect its change has on both products' prices in period $t+1$. This total derivative, shown in equation (33), multiplies the mark-up for product $j$ in period $t+1$. This term captures the expected change in future revenue due to demand dynamics arising from the price change of product $j$.
Second, there is an additional expected revenue impact that arises due to the effect that a change in the current price of product $j$ has on the forecast price of product $j$ in the next period. This derivative multiplies forecasted demand for product $j$ in period $t+1$. With only two periods, this partial derivative can be equivalently written once again as a total derivative. This substitution has been made in equations (33) and (34).
If we were to consider the problem of pricing over a horizon of three, rather two periods, the final term in equation (34) is shown in equation (35). It includes all direct and indirect effects of changes in current prices on future prices. That is, it is product of the total derivative of the effect of current price on the price of product $j$ in period $t+2$

$$
\begin{equation*}
\delta \mathbb{E} \underbrace{\left(\frac{\partial p_{j, t+2}}{\partial p_{j, t+1}} \frac{\partial p_{j t+1}}{\partial p_{j t}}+\frac{\partial p_{j, t+2}}{\partial p_{k, t+1}} \frac{\partial p_{k, t+1}}{\partial p_{j t}}+\frac{\partial p_{j, t+2}}{\partial p_{j t}}\right)}_{=\frac{d p_{j, t+2}}{d p_{j} t}} q_{j, t+2}=\delta \mathbb{E} \frac{d p_{j, t+2}}{d p_{j t}} q_{j, t+2} \tag{35}
\end{equation*}
$$

Next consider the single product duopoly dynamic pricing problem over a $H$-period horizon. In this case firm $n$ maximises the expected discounted sum of per-period profits for $H$-periods ahead

$$
\begin{equation*}
\pi_{n t}^{N P V}=\left(p_{j t}-c_{j t}\right) q_{j}\left(\boldsymbol{p}_{t}, \boldsymbol{\omega}_{t}\right)+\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t}\left(p_{j h}-c_{j h}\right) q_{j}\left(\boldsymbol{p}_{h}, \boldsymbol{\omega}_{h}\right) \tag{36}
\end{equation*}
$$

Further, let $\boldsymbol{\Delta}_{h}^{q}$ and $\boldsymbol{\Delta}_{h}^{p}$ be the matrices of the current and inter-temporal derivatives of expected future demand and price forecasts in period $h=t, t+1, \ldots, H$ with respect to change in current prices. The ( $j, k$ )-th elements of these matrices are given by equations (4) and (5).
Extending the two period result above, the optimal price of product $j$ in period $t$ of the $H$-period horizon problem satisfies the first order condition

$$
\begin{equation*}
q_{j t}+\Delta_{j j t}^{q} m_{j t}+\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \Delta_{j j h}^{q} m_{j h}+\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \Delta_{j j h}^{p} q_{j h}=0 \tag{37}
\end{equation*}
$$

as required.

## B.2. Experience Based Equilibrium

For agents' behaviour to meet MPE's rationality conditions in a storable good industry with forward-looking consumers and firms, there are three conditions that must hold for all possible realisations of the state vector. First, firms' beliefs over rivals' strategies coincide with their optimal price strategies. Second, they must also hold correct beliefs over forecasting technologies used by forward-looking consumers. Finally, a MPE also requires that consumers' forecasting technology is consistent with firms' equilibrium price strategies.
However, the demands that the MPE conditions place on the cognitive and physical resources of firms and consumers are arguably prohibitive. For example, one requirement implicit in these conditions is that all agents know the identity of the manufacturing firm for each product. While firms know the identity of manufacturers for each product, it is not obvious that consumers do. If not, the MPE condition that requires consistency between firms' equilibrium price strategies and consumers' price forecasts arguably requires more sophistication and cognitive capacity than is available to them (or at least that they would optimally choose to allocate).
As highlighted above, firms also may not have the capacity to compute the strategies that meet MPE conditions. Even with a handful of products - and therefore a small state space processing and evaluating the value functions associated with complicated price strategies is computationally challenging. When, as is common in many storable good industries, the choice set is high-dimensional (i.e. $J \approx 100$ ) the state space becomes large, then the computation of strategies satisfying MPE rationality conditions is arguably infeasible.

## B.3. Dynamic diverted value ratios

Below I show that the dynamic diverted value ratio is the product of a term containing elements of $\Psi_{t}$ and its static counterpart.

$$
\begin{align*}
D D R_{j k t}^{\pi} & :=-\frac{\mathbb{E} \sum_{h=t}^{H} \delta^{h-t} \Delta_{j k h}^{q} m_{k h}}{\mathbb{E} \sum_{h=t}^{H} \delta^{h-t} \Delta_{j j h}^{q} m_{j h}}  \tag{38}\\
& =-\frac{\Delta_{j k t}^{q} m_{k t}+\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \Delta_{j k h}^{q} m_{k h}}{\Delta_{j j t}^{q} m_{j t}+\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \Delta_{j j h}^{q} m_{j h}} \\
& =-\frac{\left(\Delta_{j k t}^{q} m_{k t}+\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \Delta_{j k h}^{q} m_{k h}\right) \frac{\Delta_{\Delta_{k t} m_{k t}}^{\Delta_{j k t} m_{k t}}}{\left(\Delta_{j j t}^{q} m_{j t}+\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \Delta_{j j h}^{q} m_{j h}\right) \frac{\Delta_{j j}^{q} m_{j t}}{\Delta_{j j t}^{\prime} m_{j t}}}}{}  \tag{39}\\
= & -\frac{\left(\Delta_{j k t}^{q} m_{k t}+\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \Delta_{j k h}^{q} m_{k h}\right) / \Delta_{j k t}^{q} m_{k t}}{\left(\Delta_{j j t}^{q} m_{j t}+\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \Delta_{j j h}^{q} m_{j h}\right) / \Delta_{j j t}^{q} m_{j t}} \times \frac{\Delta_{j k t}^{q} m_{k t}}{\Delta_{j j t}^{q} m_{j t}}  \tag{40}\\
& =\frac{1+\Psi_{j k t}}{1+\Psi_{j j t}} D R_{j k t}^{\pi} \tag{41}
\end{align*}
$$

as required.

## C. Estimation

This appendix contains additional details of the three step estimation procedure applied to the UK laundry detergent industry. It contains supplementary information on the estimation of the price process and the static nested logit demand model, respectively. Finally, the results of the sensitivity analysis to the baseline model estimating dynamic correction factors are presented.

## C.1. Step 1: Price forecasting model

The application to the UK laundry detergent industry presents results for both open-loop and closed-loop price forecast models. This section contains additional details of aspects of the a price forecasting models estimated in step one of the estimation procedure.

Hyper-parameters The minimum length of the regime window is set to 52 weeks. With 562 weeks in the sample, the corresponding trimming parameter is approximately 0.1 - in line with Baltagi et al. (2021) recommendation when serial correlation might be present. The HAC estimator of covariance matrix uses a Bartlett kernel with a bandwidth of $2 \times T^{1 / 5}$.
A maximum of eight price factors are allowed in each period. The optimal number of factors in each regime is chosen to minimise the residual sum of squares plus the $I C_{2}$ penalty term defined in Bai and Ng (2002). The optimal number of lags is chosen using the Bayes Information Criteria.

Implementation One technical problem is that prices are not observed in all weeks. To allow for missing price data, the factor model is fitted the Julia package (TSVD - Truncated Singular Value Decomposition) using a 'thin' singular value decomposition with $R$ factors applied to the a sparse matrix containing only observed prices implementing the method developed by Larsen (1998).

A second issue is that the response surfaces in Bai and Perron (2003) appear to poorly approximate critical values when evaluated far outside of the domain of simulated critical values used to estimate them. This problem arises because because the number of regressors in a vector auto-regressions increases quadratically in $R$. Therefore, at even moderate values of $R$ the quality of the approximation to critical values using the response surface can quickly degrade (i.e. they produce large negative critical values). To consider these cases, critical values are extrapolated from 'stable' tabulated critical values using a 'KingFit' procedure in the Julia package CurveFit.

Structural break tests Table 6 contains the result of the iterative testing procedure. The upper panel shows three tests - supF, UDmax, WDmax - to test for the presence of structural breaks under the null of no structural breaks. All tests rejects the null hypothesis at the 1 percent level.
The bottom panel shows the results of the iterative test procedure. In each case, the null of $l$ structural breaks is tested against the alternative of $l+1$ structural breaks. The test continues until the null cannot be rejected.
The first column in Table 6 contains the number of structural breaks under the alternative hypothesis (i.e. $l+1$ ). The second column is the number of regressors in Bai and Perron (2003)

Table 6: Structural Break Tests

| Tests for at least one structural break |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  |  | Critical Values |  |  |
| Breaks | Test Stat. | $10 \%$ | $5 \%$ | $1 \%$ |
| $\sup F(1 \mid 0)$ | 2.52 | 0.8 | 0.83 | 1.05 |
| UDmax | 2.52 | 1.09 | 1.13 | 1.35 |
| WDmax: $10 \%$ | 2.82 | 1.12 | - | - |
| WDmax: $5 \%$ | 3.00 | - | 1.17 | - |
| WDmax: $1 \%$ | 3.05 | - | - | 1.42 |


| Iterative structural break tests |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Critical Values |  |  |
| $l+1$ | q | $\sup F_{N T}(l+1 \mid l)$ | $10 \%$ | $5 \%$ | $1 \%$ |
| 1 | 3 | 30.75 | 15.68 | 17.54 | 21.35 |
| 2 | 6 | 48.55 | 22.75 | 24.85 | 28.74 |
| 3 | 6 | 48.55 | 23.26 | 25.35 | 29.17 |
| 4 | 6 | 48.55 | 23.71 | 25.79 | 29.55 |
| 5 | 6 | 39.22 | 24.13 | 26.19 | 29.89 |
| 6 | 3 | 38.38 | 17.96 | 19.77 | 23.28 |
| $7^{\dagger}$ | 36 | 30.21 | 64.70 | 74.40 | 80.47 |

$\dagger$ Critical values for the number of degrees of freedom lie outside tabulated values in Bai and Perron (2003). Reported critical values are extrapolated using a 'KingFit' procedure in the Julia package CurveFit.
critical values response surfaces. ${ }^{61}$ The test statistic and critical values at the ten, five and one percent level comprise the remaining columns.

## C.2. Step 2: Estimation of the static demand model

This section contains additional details of aspects of the a static discrete choice demand model of UK laundry detergent estimated in step two of the estimation procedure.

Time aggregation With the goal of measuring long-run demand responses, it may be tempting to apply a static demand model to a coarser time-partition of the market data. For

[^32]example, estimating a static demand model on data aggregated to monthly frequency then using its demand derivatives to approximation to long-run price elasticities. Unfortunately, given complex promotional pricing patterns and corresponding dynamic demand responses, it is not clear how the resulting demand derivatives relate to long-run substitution patterns.
Another potential concern is that any bias may be compounded when the output of a static demand model estimated on time aggregated data is used as in input into a mis-specified, timeaggregated static Nash-Bertrand supply model. The corresponding policy simulations inherit and potentially magnify the (unknown) biases in its inputs. Even if the misspecified supply side model were considered to be a useful approximation to long-run industry outcomes, the correct inputs would be those derived from quantity simulations using a dynamic demand model, not those from misspecified static model applied to overly time aggregated data.

Choice Sets The choice sets are constructed using all purchases at stores at the major UK supermarket between 1st January 2002 and 31st October 2012. The purchase diary data only records when and where a product is purchased. As a result, due to sampling variation some products are not observed in the data even though they are available for purchase. This makes it difficult to determine which products were available at the point of purchase. To address this, I assume that a product is eligible for the choice set if a purchase is observed in the same calendar month as the purchased product. Further, if a product is purchased in a promoted bundle of two units, the opportunity to purchase two units of product is included in the choice set. If there is no observed purchase of two units, it is assumed it is not available as a promotional bundle and the total price is twice that of a single unit of the product.
This approach leads to the inclusion of products in the choice set without observed prices. To remedy this these products' prices need to be imputed. ${ }^{62}$ Where necessary, the DFM from in step one of the estimation procedure is used to impute prices.

Control function The possible source of endogeneity is reflected by the correlation of product-specific unobservables in the choice model, $\xi_{j t}$, and the unobservable component of the auxiliary price equation, $v_{j t}$. When observed prices are regressed on product characteristics and instruments, the estimated residuals from this regression contains the correlation between $p_{j t}$ and $\xi_{j t}$. As a result, the estimated residuals, $\hat{v}_{j t}:=p_{j t}-\mathbb{E}\left[p_{j t} \mid \boldsymbol{x}_{j}, \boldsymbol{z}_{j t}\right]$, are used to construct a control function $C F\left(\hat{v}_{j t} ; \rho\right)=\rho \hat{v}_{j t}$.
When this control function is added to the choice model it conditions out that part of the prices correlated with $\xi_{j t}$. As a result, it resolves the endogeneity that arises from the unobserved product-market terms, $\xi_{j t}$. The unobserved component of demand is now independent of price, $\epsilon_{i j t} \perp p_{j t} \mid C F\left(\hat{v}_{j t} ; \rho\right)$, and $\epsilon_{i j t}={ }^{d} \varepsilon \stackrel{i i d}{\sim} G E V .{ }^{63}$
The results of estimating the auxiliary equation are shown in Table 7 .

[^33]Table 7: Control Function: parameter estimates

| Variables | $\theta$ | se |
| :--- | :---: | :---: |
| Intercept | 1.589 | 0.058 |
| Characteristics |  |  |
| Washes | 14.503 | 0.036 |
| Dosage | 0.678 | 0.102 |
| Dosage x Liquids | 0.953 | 0.135 |
| Instruments |  |  |
| Num. own similar prods | -0.038 | 0.004 |
| Num. rival's similar prods | -0.040 | 0.003 |
| Detergent Fixed Effects | Yes |  |
| Year Fixed Effects | Yes |  |
| N | 59,035 |  |
| $R^{2}$ | 0.797 |  |

## C.3. Step 3: Estimation $\theta$ in the UK laundry detergent industry

This section provides further details on the estimation of $\boldsymbol{\theta}$ under the baseline model and presents sensitivity analysis referred to in the main text.

Incorporating VAT When VAT is included in the model, the net price received by the firm is the market price divided by one plus the VAT level in that period. The resulting modification to the firm's objective function is

$$
\begin{equation*}
\pi_{n t}^{N P V}=\mathbb{E} \sum_{h=t}^{H} \sum_{j \in \mathcal{J}_{n}} \delta^{h-t}\left(\frac{p_{j h}}{1+V A T_{h}}-m c_{j h}\right) q_{j}\left(\boldsymbol{p}_{h}, \boldsymbol{\omega}_{h}\right) \tag{43}
\end{equation*}
$$

Therefore, with VAT the first order conditions are modified as follows

$$
\begin{equation*}
\frac{\boldsymbol{q}_{n t}}{1+V A T_{t}}+\boldsymbol{\Delta}_{n t}^{q} \widetilde{\boldsymbol{m}}_{t}+\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \boldsymbol{\Delta}_{n h}^{q} \widetilde{\boldsymbol{m}}_{h}+\frac{\mathbb{E} \sum_{h=t+1}^{H} \delta^{h-t} \boldsymbol{\Delta}_{n h}^{p} \boldsymbol{q}_{h}}{1+V A T_{t}}=\mathbf{0} \tag{44}
\end{equation*}
$$

where $\boldsymbol{w}_{t}=\frac{\boldsymbol{p}_{t}}{1+V A T_{t}}$ and $\widetilde{\boldsymbol{m}}_{t}:=\boldsymbol{w}_{t}-\boldsymbol{m} \boldsymbol{c}_{t}$.
Therefore, the model's margin equation over $T$ periods incorporating VAT is

$$
\begin{equation*}
\mu\left(\theta_{n}^{o w n}, \theta_{n}^{c r o s s}\right):=-\frac{\sum_{t=1}^{T} \boldsymbol{q}_{n t}^{\top}\left(\widehat{\boldsymbol{\Delta}}_{n t}^{q} \odot \boldsymbol{\theta}_{n}\right)^{-1}\left(\boldsymbol{q}_{t}+\boldsymbol{\Gamma}_{n t} \overline{\boldsymbol{q}}\right) \oslash\left(1+V A T_{t}\right)}{\sum_{t=1}^{T} \boldsymbol{q}_{n t}^{\top} \boldsymbol{w}_{t}} \tag{45}
\end{equation*}
$$

where " $\oslash$ " denotes element-wise division.

Sensitivity analysis In addition to the baseline model, I consider two scenarios to test the findings of the policy analysis to alternative model calibrations. The first scenario (S1) is the same as the baseline model but an open-loop price forecasting model is used. This scenario reflects the behaviour of a boundedly rational firm with significant organisational frictions that limit feedback between price setting and forecasting process. That is, $\boldsymbol{\Delta}_{t}^{p}=\mathbf{0}$.
The second scenario (S2) is the baseline model with closed-loop price forecasting model with an optimisation window equal to the mean inter-purchase duration. The values of $\bar{\theta}_{n}^{\text {cross }}$ under this scenario are larger than the baseline model and are shown in the last row of Table 5 . It reflects a situation where a consumer's inter-temporal substitution is a larger component of overall substitution. In this scenario, households accelerate purchases in response to a promotion that would have happened further in to the future.
The results of scenarios are summarised by two charts: (i) the bounds $\left[\underline{\theta}_{n}^{\text {own }}, \bar{\theta}_{n}^{\text {own }}\right]$ under each scenario; and (ii) dGUPPI bounds under each scenario. Figure 5 shows that allowing closedloop forecasting models with longer optimisation windows reduces the adjustment needed match margins. This is especially pronounced for firm A. This is because firm A has more products than firm B. This is also consistent with promotional pricing being used as an inter-temporal price discrimination tools to increase market power.
However, after 2009 the adjustments to short-run own-price demand responses overlap to a greater extent for all three scenarios and for both companies. This coincides with the period where discounts are deeper and more frequent. Both firms' products are also aggressively compacted and sold in larger sizes. These observations are consistent with both firms being less able to use inter-temporal price discrimination strategies in this period to increase market power. This in turn suggests that promotional pricing strategies may be more focused on consumer acquisition than inter-temporal price discrimination.
Figure 6 shows the dGUPPI ranges under the three scenarios from 2002 to 2012. The solid lines plot the lower bound of dGUPPI and the dashed lines the corresponding upper bounds.
For $d G U P P I_{A E}$, relative to the two other scenarios the baseline model produces conservative estimates for the range of dGUPPI. Without adjustments for market power arising from intertemporal price discrimination, the open-loop forecast model has a higher lower bound than the baseline model. In contrast, both the baseline and the closed-loop model scenario with a longer optimisation window have similar lower bounds. As expected, given a higher value the

Figure 5: $\theta_{n}^{\text {own }} \in \Theta_{n}^{\star}$ : Sensitivity analysis with $\bar{\theta}_{n}^{\text {cross }}$ calibrated with the baseline model's intertemporal substitution

Firm A


Firm B

upper bound on $\theta_{n}^{\text {cross }}$, the closed-loop sensitivity has a higher upper bound than the baseline $d G U P P I_{A E}$. These patterns are broadly similar for the $d G U P P I_{E A}$ - though less pronounced. Overall, the qualitative conclusion of the baseline model is unchanged in all scenarios. Namely, in the past consumers perceive brands A and E to be close enough substitutes to flag the potential for harmful unilateral effects to arise if brand E is acquired by the firm producing brand A .

Figure 6: dGUPPI: Sensitivity analysis with $\bar{\theta}_{n}^{\text {cross }}$ calibrated with baseline model's intertemporal substitution



[^0]:    Acknowledgments: An older version of this paper was previously circulated under the title "Approximating demand dynamics in storable goods industries". I would like to thank Lars Nesheim, Michela Tincani, Pedro Mira, Mateusz Mysliwski, Marleen Marra, Johannes Schneider, and participants at the Barcelona Summer Economic Forum 2023 and BECCLE 2023 for helpful comments and suggestions. I would also like to thank the Economic Social Research Council (ESRC) for funding this research. I also gratefully acknowledge financial support from the European Research Council (ERC) under ERC-2009-AdG grant agreement 249529 and the Agencia Estatal de Investigación (Spain), grant PID2019-10789GB-I00. Data supplied by Kantar Worldpanel. The use of Kantar Worldpanel data in this work does not imply the endorsement of Kantar Worldpanel in relation to the interpretation or analysis of the data.
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[^1]:    ${ }^{1}$ Dynamic diverted value ratios measure the combined effect on the present value of profit earned from immediate and inter-temporal consumer responses to deep, temporary discounts that characterise storable good pricing.

[^2]:    ${ }^{2}$ The use of margins calculated from internal accounts is common in merger screening tools. Notwithstanding the well-documented conceptual differences between the economic and accounting margins, antitrust authorities have placed evidentiary weight on empirical analysis based on firms' internal margin data (see footnote 53). See Pittman et al. (2009) on the use of accounting cost information as an input into antitrust policy. An alternative approach is to use the method proposed by De Loecker et al. (2020) to estimate mark-ups from published accounts.
    ${ }^{3}$ Chen et al. (2008) show that the omission of demand and supply dynamics for durable goods leads to biased estimation of demand elasticities.

[^3]:    ${ }^{4}$ Policy analysts may also have access to internal accounting data to produce more refined margin estimates.
    ${ }^{5}$ This approach to the closed-loop forecasting scenario may serve a sensitivity if the analyst has access to internal forecasting models.
    ${ }^{6} \mathrm{~A}$ bias correction is required because the short-run demand derivatives estimated using a static demand model are biased estimators of the true short-run demand responses when they incorrectly omit inventory holdings. If the effect of omitting inventory on the bias of marginal utility of income is the same for "switchers" and "brand loyal" consumers, then the bias scales the static demand derivatives by the same multiplicative factor (i.e. this holds for static discrete choice demand models with homogeneous marginal utility of income (i.e. logit and nested logit)).

[^4]:    ${ }^{7}$ Short-run diverted value ratios have also been called "profit retention ratios".
    ${ }^{8}$ Dosage per wash is the physical amount of detergent needed per wash.

[^5]:    ${ }^{9}$ More distantly related are Goettler and Gordon (2011) and Carranza (2010). Both estimate dynamic demand and supply models in durable good industries.
    ${ }^{10}$ This pricing policy follows an undertaking in 2001 following a market investigation by the UK Competition Commission into the groceries industry in 2000.

[^6]:    ${ }^{11}$ Brands are not attributed to firms for confidentiality reasons.

[^7]:    ${ }^{12}$ The UK laundry detergent industry differs in this respect from the one studied by Hendel and Nevo (2006). They restrict attention to powder products and examine brand choice conditional on size choice from a small number of discrete sizes: $16 \mathrm{oz}, 32 \mathrm{oz}, 64 \mathrm{oz}, 96 \mathrm{oz}$, and 128 oz . Erdem et al. (2003) also focus on only five different weight choices in the US Ketchup market in their dynamic demand estimation.
    ${ }^{13} \mathrm{~A}$ more in depth discussion of the product compaction process and its impact on consumer demand and firm pricing is provided in Appendix A.

[^8]:    ${ }^{14}$ For expositional simplicity I abstract away from the possibility of there being variations in the number of goods available for sale in each time period.

[^9]:    ${ }^{15}$ The window for consuming a storable good and the related storage costs depend on their characteristics. For example, some storable goods, such as yoghurts and other dairy products, can be stored for three to four weeks in refrigerated conditions. Others, such as laundry detergent, are non-perishable but may occupy a relatively large portion of storage space.
    ${ }^{16}$ For example, Kunz et al. (2023) publish Zalando's empirical dynamic demand function for clothing used since 2019. They describe in detail how large data sets containing recent market outcomes are combined machine learning and high-dimensional time-series techniques to make near-term demand forecasts and aid internal pricing decisions.
    ${ }^{17}$ Specifically, Nair (2019) states that this 'may not be unreasonable' in stable industries where firms have ready access to large amounts of data. In line with this perspective, laundry detergent firms are large multinational

[^10]:    firms with extensive industry experience, have access to detailed historical information on market outcomes, and intensively research and study consumer preferences.
    ${ }^{18}$ This is the approach taken in much of the empirical IO and marketing literature analysing fast-moving consumer goods industries. Selected examples relevant to storable goods include Hendel and Nevo (2013); Pavlidis and Ellickson (2017); Myśliwski et al. (2020); Nevo (1998, 2001); Slade (2004). Moreover, the assumption that manufacturer set prices with a 'passive' retailer has also been adopted in antitrust cases involving storable goods. For example, after a detailed review of internal documents, the DG COMP adopt this assumption in COMP/M.56858 - Unilever/Sara Lee - a merger of storable personal care products ("Technical Annex: Demand estimation and merger simulation", p-60, pp348).

[^11]:    ${ }^{19}$ See Powell (2011); Bertsekas (2011) for detailed overview of ADP.
    ${ }^{20}$ This is also known as the receding horizon procedure or, in engineering, model predictive control. It also describes online components of many reinforcement learning algorithms successfully applied to Backgammon, Chess and Go. See Rust (2019), Igami (2020) and Iskhakov et al. (2020) for a discussion of these methods.

[^12]:    ${ }^{21}$ See Appendix B to see how total derivatives capture both the direct and indirect effect that price changes today have on future quantities and prices.

[^13]:    ${ }^{22}$ See Appendix B. 3 for a derivation.

[^14]:    ${ }^{23}$ Storable good industries with dynamic demand contain sell $J \geq 100$ products in each week sold across multiple retail outlets.
    ${ }^{24}$ For example, Hortaçsu et al. (2023) document how organisational frictions require a large US airline to make post-hoc adjustments to forecasted demand inputs so that the revenue management systems implement the desired pricing strategy.

[^15]:    ${ }^{25} \mathrm{MPE}$ is used by Chen et al. (2008) and Goettler and Gordon (2011) in their studies of dynamic demand and supply systems for durable goods. In contrast, in his study of the camera industry Carranza (2010) assumes that firms behave monopolistically and ignore current and future strategic interactions when optimising profits.
    ${ }^{26}$ Appendix B. 2 discusses the practical challenges for firms and consumers to meet the rationality requirements of MPE in more detail.

[^16]:    ${ }^{27}$ Another possibility is to use a linear-in-prices $\operatorname{VAR}(\kappa)$ approach to model the price process. While simple to use, it is ill suited to high-dimensional applications. The online appendix discusses this issue in more depth and provides more detail on the DFM.

[^17]:     of price factors.
    ${ }^{29}$ The choice of $R$ and $\kappa$ are based on information criteria. This is discussed further in Appendix C.
    ${ }^{30}$ See the online appendix for a derivation of price derivatives.
    ${ }^{31}$ The validity of this assumption can be empirically examined by evaluating the covariances of $\widehat{\Delta}_{j k t+h}^{p}$ and $\widehat{q}_{k}\left(\boldsymbol{p}_{t+h}\right)$ for $h=1, \ldots, H$ over the $T$ periods for $k \in \mathcal{J}$. If empirical covariances are not deemed to be sufficiently close to zero, the empirical covariance for each ( $j, k, t, h$ ) can be added to equation (19).

[^18]:    ${ }^{32}$ When households observe a temporary sale, those with sufficiently low existing inventory may accelerate purchases and take advantage of discounted prices. As a result, inventories tend to be positively correlated with high prices that follow the end of the temporary sale periods. Therefore, when prices exhibit positive serial correlation (i.e. they are low today, but expected to be high in the next period), omitting inventory negatively biases the static demand's price coefficient. In turn, fortifying the bias due to the exclusion of demand dynamics from the static demand model.

[^19]:    ${ }^{33}$ For expositional purposes I will focus on firm n's margin. More generally, the empirical approach described in this section can be applied to margins earned on sales of any group of products (i.e. brand level margins).

[^20]:    34" $\oslash$ " denotes element-wise division.
    ${ }^{35}$ For empirical support for this assumption see Table 10 in Erdem et al (2003) simulated dynamic substitution patterns in the US Ketchup industry. The relative dynamic responses of non-promoted goods appear approximately symmetric over time relative to the contemporaneous response.

[^21]:    ${ }^{36}$ Diversion ratios enumerated using volume weights as described Domencich and McFadden (1975).
    ${ }^{37}$ The open-loop forecast model is used in sensitivity analysis. In practice, with access to internal documents, a policy analyst may know the forecasting model used by firms. In this case, the open and closed loop models may serve as sensitivities.

[^22]:    ${ }^{38}$ The chosen hyper-parameters and detailed breakdown of the structural break tests are located in Appendix C.1.
    ${ }^{39}$ The discussion in Appendix A contains a more detailed description of the changes in product dosages, sizes and pricing behaviour over the sample period.

[^23]:    ${ }^{40}$ See Appendix C. 2 for more detailed description of the construction of the choice sets.
    ${ }^{41}$ Given that the focus of this paper is the recovery of dynamic diverted value ratios, the details of the static demand estimation procedure is discussed in Appendix C.2.

[^24]:    ${ }^{42}$ Estimates of the control function are provided in Annex C.2.
    ${ }^{43}$ The proxy for household income is annual average weekly expenditure on all groceries.
    ${ }^{44}$ To calculate equivalent adults, I use the OECD-modified equivalence scale.

[^25]:    ${ }^{45}$ In their dynamic demand estimation of the UK ketchup market, Erdem et al. (2003) find exponentially declining purchases in response to a temporary one period price cut.
    ${ }^{46}$ Table 5 shows the values of $\bar{\theta}_{n}^{\text {cross }}$ under the median and mean inter-purchase duration. Appendix C shows how results are affected when the scenario when the optimisation window equals the mean inter-purchase duration.
    ${ }^{47}$ From the beginning of the sample until 30th November 2008 UK VAT was $17.5 \%$. From 1st December 2008 to 30th December 2009 it was cut to $15 \%$. It was returned to $17.5 \%$ from 1st January 2010 until 3rd January 2011. Since 4th January 2011 UK VAT has been $20 \%$.

[^26]:    ${ }^{48}$ The financial year for firm B starts midway through the year. As such, the annual report margins are adjusted to match calendar years in the data. Figures are omitted for confidentiality reasons.
    ${ }^{49}$ The first two periods are dropped in 2002 because two lags are required to produce estimates $\boldsymbol{\Gamma}_{n t} \overline{\boldsymbol{q}}$.
    ${ }^{50}$ This is in line with Hendel and Nevo (2006) finding that short-run own price elasticity for US laundry detergent purchases in the early 1990s overstate their long run counterparts by approximately 30 percent.

[^27]:    ${ }^{51}$ The ratio of $\theta_{n}^{\text {cross }}$ to $\theta_{n}^{\text {own }}$ is increasing when products in $\mathcal{J}_{n}$ are perceived to be closer substitutes. The ratio of $\theta_{A}^{\text {cross }}$ to $\theta_{A}^{\text {own }}$ falls 38 percent from 0.071 in 2002 to 0.044 by 2011.
    ${ }^{52}$ The ratio of $\theta_{B}^{A}$ ross to $\theta_{B}^{\text {own }}$ for $\left(\theta_{B}^{\text {own }}, \theta_{B}^{\text {cross }}\right) \in \Theta_{B}^{\star}$ falls 35 percent from 0.049 in 2002 to 0.032 by 2012.

[^28]:    ${ }^{53}$ GUPPI and other price pressure tests have been used in many recent merger cases. For example, in the UK, these include Sainsbury's/Asda (2019), Tesco/Booker (2017), Ladbrokes/Coral (2016) and Cineworld/Showcase Cinema (2013). Examples in the EU and USA include Austria / Orange Austria (2012) and Dollar Tree/Family Dollar (2015), respectively.
    ${ }^{54}$ Moresi (2010) advocates the use of gross margins. Similar accounting measures derived from internal documents have been used in the cases listed in footnote 53 .

[^29]:    ${ }^{55}$ The potential pitfalls of assuming static demand estimation applied to time and product aggregated data is discussed at greater length in Appendix C.
    ${ }^{56}$ GUPPI is often derived from static first order conditions of two merging products. The online appendix derives dynamic GUPPI using the first order conditions from the dynamic demand and supply model from section 3.
    ${ }^{57}$ For confidentiality reason, brands cannot be explicitly assigned to firms.

[^30]:    ${ }^{58}$ Five percent (or higher) might be used because it is often used as the floor for a significant non-transitory increase in price for market definition purposes. However, for goods that comprise a large fraction of consumption, competition authorities have used more stringent GUPPI thresholds. For example in the investigation into the proposed Asda/Sainsbury's merger the UK Competition Markets Authority used a 2.75 percent GUPPI threshold; a 1.5 percent baseline with a 1.25 percent efficiencies allowance to assess local market competition. See Competition Markets Authority (2019) "Anticipated merger between J Sainsbury PLC and Asda Group Ltd: Final Report".

[^31]:    ${ }^{59}$ The UK laundry detergent industry differs in this respect from the one studied by Hendel and Nevo (2006). They restrict attention to powder products and examine brand choice conditional on size choice from a small number of discrete sizes: $16 \mathrm{oz}, 32 \mathrm{oz}, 64 \mathrm{oz}, 96 \mathrm{oz}$, and 128 oz . Erdem et al. (2003) also focus on only five different weight choices in the US Ketchup market in their dynamic demand estimation.
    ${ }^{60}$ To avoid complications arising from correcting for changing dosages, the number of washes is used to measure pack size and consumption.

[^32]:    ${ }^{61}$ Under the null of $l$ structural breaks, $q=R_{i}\left(R_{i}+1\right) / 2$ where $R_{i}$ is the number latent price factors associated with the largest value of the test statistic for across $l+1$ regimes.

[^33]:    ${ }^{62}$ Approximately 7 percent of observations are imputed.
    ${ }^{63}$ Where $={ }^{d}$ is defined as "follows the same distribution as".

